The ratio of the real part to the imaginary part of the elementary amplitude for the scattering of hadrons by nuclei

O. D. Dal'karov and V. A. Karmanov
P. N. Lebedev Physics Institute, Academy of Sciences of the USSR, Moscow

(Submitted 15 November 1984)
Pis'ma Zh. Eksp. Teor. Fiz. **41**, No. 1, 47–50 (10 January 1985)

In the eikonal approximation, the absolute value in the minima of the differential cross section for scattering of hadrons by nuclei is determined completely by the ratio of the real part to the imaginary part of the elementary amplitude. The method proposed here is used to determine this ratio for the $\bar{p}N$ scattering amplitude from the most recent data on the scattering of antiprotons by nuclei.

Dal'karov and Karmanov[1] have shown that scattering of antiprotons by nuclei is described well by the Glauber approximation down to very low energies. The theoretical predictions were compared with experimental data on the elastic and inelastic scattering (with the excitation of the residual nucleus) of $\bar{p}$ by the ^{12}C nucleus. The experimental data were obtained at an energy of 46.8 MeV on the LEAR antiproton storage ring.[2] New data obtained at other energies have recently been reported. These data, as will be shown below, can also be described well by the Glauber approximation. In this letter we show that because of these circumstances, the scattering of antiprotons by nuclei can be conveniently used as an independent method of determining quite accurately over a broad energy range the most ambiguous parameter of the $\bar{p}N$ scattering amplitude—the ratio $\epsilon = \mathrm{Re} f_N(0)/\mathrm{Im} f_N(0)$, where $f_N(0)$ is the amplitude of the $\bar{p}_N$ scattering through zero angle (as usual, the scattering amplitude is parametrized as $f_N(q) = [k\sigma(i+\epsilon)/4\pi]\, e^{(-1/2)Bq^2}$, where q is the momentum transfer, σ is the total interaction cross section, and k is the momentum of the incident antiproton.

The method proposed here is based on the fact that the differential cross sections for scattering of $\bar{p}$ by a nucleus in the diffraction minima are highly sensitive to ϵ. The solid curve in Fig. 1 represents the elastic cross section for scattering of antiprotons by the ^{12}C nucleus at 180 MeV. Also shown for comparison in this figure are the new LEAR data of Ref. 3. The dotted curve shows the behavior of the square of the real part $[\mathrm{Re}F(q)]^2$ of the antiproton-nucleus amplitude $F(q)$ and the dashed curve shows the behavior of the square of the imaginary part $[\mathrm{Im}F(q)]^2$ of the antiproton-nucleus amplitude. The sum of these curves is the cross section represented by the solid curve. We see that the cross section is determined by $\mathrm{Im}F(q)$ everywhere except at the minima; i.e., $[\mathrm{Re}F(0)]^2 \ll [\mathrm{Im}F(0)]^2$. The opposite situation occurs at the minima of the cross section determined by the minima of $[\mathrm{Im}F(q)]^2$: The cross section is the same as the value of $[\mathrm{Re}F(q)]^2$, which is highly sensitive to ϵ. Figure 1 shows two calculations of $[\mathrm{Re}F(q)]^2$ corresponding to $\epsilon = 0.3$ (LEAR data of Ref. 4, obtained at 180 MeV) and $\epsilon = 0.04$ (the data of Ref. 5, obtained at the same energy; see also Fig. 3). These curves have a ratio of 58 at $\theta = 23°$, in approximate agreement with the ratio of the squares of ϵ: $(0.3/0.04)^2 = 56$. This shows that the ratio $d\sigma/d\Omega$ at the minima is highly sensitive

 0021-3640/85/010058-05$01.00

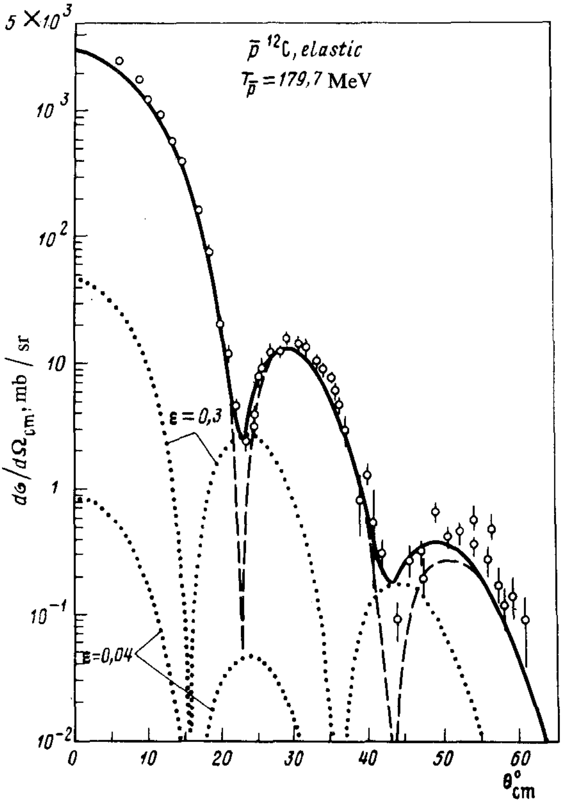

FIG. 1. Differential cross section for elastic scattering of $\bar{p}$ by the ^{12}C nucleus. Dotted curves—the square of the real part of the $\bar{p}$ – ^{12}C scattering amplitude; dashed curve—the square of the imaginary part; solid curve—the sum of the real and imaginary parts for $\epsilon = 0.3$. The experimental data were taken from Ref. 3.

to ϵ, where $d\sigma/d\Omega$ is nearly proportional to ϵ^2. The sensitivity of the cross section to ϵ is low everywhere on the curve except at the minima.

We wish to emphasize that the minima of the cross section are the maxima of $[\mathrm{Re}F(q)]^2$, a highly favorable situation. For this reason, we can expect that the other effects (the corrections to the Glauber approximation, for example) may prove to be weak. For the same reason, the variation of ϵ does not lead to a displacement of the minima, so that the ϵ dependence of the cross section cannot be simulated by varying the nuclear radius.

These properties of the cross section, which have been illustrated by numerical calculations, are very general in nature (they do not depend on the type of the incident hadron, on its energy, or on the target nucleus)[1)] and can easily be found analytically. In terms of the Glauber approach, we have

$$F(q) = k \int_0^\infty i\Gamma(b) J_0(qb) b\, db, \qquad (1)$$

where

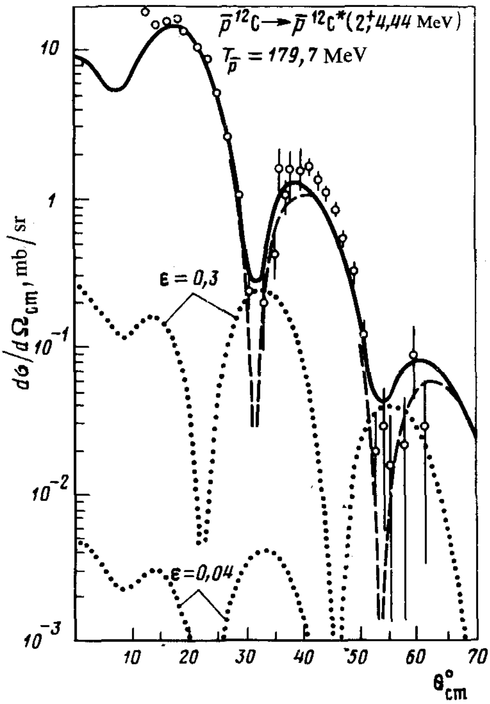

FIG. 2. Differential cross section for inelastic scattering [with the excitation of the 2^+ level (4.44 MeV)] of $\bar{p}$ by ^{12}C. The notation is the same as in Fig. 1.

$$\begin{aligned}\mathrm{Re}(i\Gamma(b)) &= -\exp(-\chi'(b))\sin(\epsilon\chi'(b)),\\ \mathrm{Im}(i\Gamma(b)) &= 1-\exp(-\chi'(b))\cos(\epsilon\chi'(b)).\end{aligned} \tag{2}$$

Here $\chi'(b)$ is a real value,

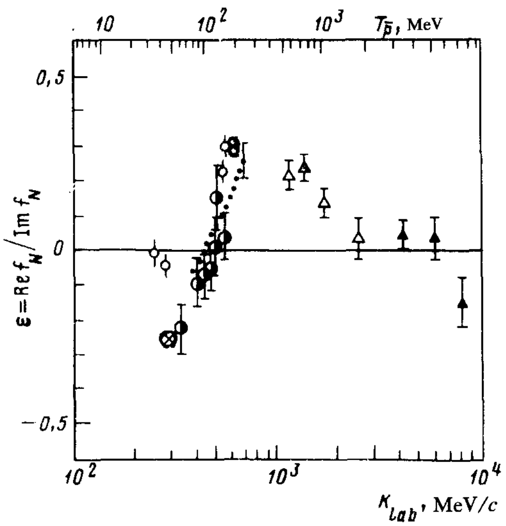

FIG. 3. The ratio ϵ of the real part to the imaginary part of the $\bar{p}N$ scattering amplitude. O—LEAR data[4]; the other data are taken from Ref. 5; ⊗—the value of ϵ found from our present calculations and from those of Ref. 1.

$$\chi'(b) = \frac{1}{8\pi^2} A\sigma \int e^{-i\mathbf{q}\mathbf{b}} e^{(-1/2)Bq^2} \Phi(\mathbf{q})\, d^2q, \tag{3}$$

where $\Phi(\mathbf{q})$ is the elastic form factor of the nucleus. The quantity $\chi'(b)$ is nonvanishing (large) inside the nucleus and rapidly approaches zero outside it. For this reason, the function $\mathrm{Im}[i\Gamma(b)]$ is nonvanishing only inside the nucleus and $\mathrm{Re}[i\Gamma(b)]$ is nonvanishing only at the nuclear surface [see Eq. (2)]. To explain the essence of this argument, we will replace $\mathrm{Im}[i\Gamma(b)]$ by a "step": $\mathrm{Im}[i\Gamma(b)] = 1$ for $b < R$ and $\mathrm{Im}[i\Gamma(b)] = 0$ for $b > R$ and we assume that $\mathrm{Re}[i\Gamma(b)]$ is a δ function: $\mathrm{Re}[i\Gamma(b)] \sim \delta(b - R)$. We then will have $\mathrm{Re}F(q) \sim J_0(qR)$ and $\mathrm{Im}F(q) \sim J_1(qR)$. Since $dJ_0(z)/dz = -J_1(z)$, the maxima of $\mathrm{Re}F(q)$ coincide with the zero values of $\mathrm{Im}F(q)$, i.e., with the minima of the cross section. In the interval (1), the region in which $\chi'(b) \sim 1$ is important. At $|\epsilon| < 1$, the sine in (2) can therefore be replaced by $\epsilon\chi'(b)$, so that $[\mathrm{Re}F(q)]^2 \sim \epsilon^2$. We see from (2) that the cross section is not sensitive to the sign of ϵ.

In the case of inelastic cross sections (with the excitation of the nuclear levels), the depth of the minima is also proportional to ϵ^2. Figure 2 shows the cross section for scattering of antiprotons by the ^{12}C nucleus at 179.7 MeV ($k = 608$ MeV/c) with the excitation of the 2^+ level (4.44 MeV), along with the new LEAR data (Ref. 3). The calculation was carried out in the approximation of a single inelastic collision (see the equations in the papers of Ref. 1). We see that the cross section in the minima is also determined by $[\mathrm{Re}F_{fi}(q)]^2$. We see from a comparison of the calculation results of $[\mathrm{Re}F_{fi}(q)]^2$ (the dotted curves) that $[\mathrm{Re}F_{fi}(q)]^2 \sim \epsilon^2$. These properties can also be easily found analytically, although the reasons for their existence are slightly different from those given in the case of elastic scattering.

The results of Refs. 4 and 5 for the value of ϵ are summarized in Fig. 3. The crosses denote the values of ϵ found by comparing the results of our present calculations and those of Ref. 1 with the nuclear data. We see that at $k = 608$ MeV/c the value $\epsilon = 0.3$ is the same as the LEAR results (Ref. 4), although a slightly smaller value, $\epsilon = 0.26$, should not be ruled out. At 300 MeV/c, our calculations[1] give the value $|\epsilon| = 0.25$, which is the same as the value found by extrapolating the results of Ref. 5. There are no LEAR data available at this momentum, while at $k = 325$ MeV/c they are at variance with the data of Ref. 5.

In summary, the analysis of antinucleon-nucleus cross sections in terms of the Glauber approximation can be used to obtain information about the parameters of $N\bar{N}$ amplitude down to very low energies. For this purpose, it is necessary to measure the antinucleon-nucleus cross sections at different energies of the antinucleon beam. Experiments of this sort will probably be carried out on the LEAR storage ring in the near future. The ratio of the real part to the imaginary part of the elementary amplitudes for the scattering of other hadrons by nucleons can be determined by accurately measuring the hadron-nucleus cross sections in the diffraction minima.

We wish to thank Prof. I. S. Shapiro for valuable critical comments and Prof. D. Garreta for graceously providing us with new experimental data obtained by his group.

[1]The fact that diffraction minima are filled in the scattering of high-energy protons by nuclei when the real part of pN scattering amplitude is taken into account was mentioned by Alkhazov *et al.*[6]

[1]O. D. Dal'karov and V. A Karmanov, Pis'ma Zh. Eksp. Teor. Fiz. **39**, 288 (1984) [JETP Lett **39**, 345 (1984)]; FIAN Preprint No. 77, 1984; Phys. Lett. **147B**, 1 (1984).
[2]D. Garreta *et al.*, Phys. Lett. **135B**, 266 (1984).
[3]D. Garreta *et al.*, Preprint CERN-EP/84-93, 1984; Preprint CERN-EP/84-114, 1984.
[4]Yh. Walcher, Seventh European Symposium on Antiproton Interactions, Durham, 9–13 July 1984.
[5]M. Cresti, L. Peruzzo, and G. Sartori, Phys. Lett. **132B**, 209 (1983).
[6]G. D. Alkhazov, S. L. Belostotsky, and A. A. Vorobyov, Phys. Rep. **42C**, 89 (1978).

Translated by S. J. Amoretty