Stochastic self-excited oscillations in low-mobility semiconductors

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The time evolution of the fluctuations in the electric field and in the free-carrier density in a low-mobility semiconductor is analyzed. If the voltage across the sample is sufficiently high and/or the sample is illuminated by sufficiently intense light of the appropriate frequency, a stationary state of the system will be unstable, and stochastic self-excited oscillations of the electric field and of the free-carrier density will set in.

The possible onset of stochastic self-excited oscillations in semiconductors with hot electrons was studied in Refs. 1 and 2. In materials with a low mobility, the heating of electrons is replaced by another factor: the field dependence of the ioniza-

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tion energy by virtue of the Poole-Frenkel' effect and the field dependence of the mobility μ, by virtue of several factors.[3] In this letter we use a model energy spectrum somewhat analogous to that proposed by Mott, Street, and Davis.[3] Specifically, we assume that the kinetics of the electron processes is dominated by defects of one type, which can be in one of three states: a positively charged state (D_+; level E_+ and density N_+), a neutral state (D_0; E_0 and N_0), and a negatively charged state (D_-; E_- and N_-). The correlation frequency, $-U$, is assumed negative; and the condition $E_+ + E_- < 2E_0$ holds. In contrast with the model of Mott, Street, and Davis, however, we consider the case in which the level E_- nevertheless lies closer to the conduction band than to the valence band, and electron exchange with the latter is ruled out (incorporating this exchange complicates the problem significantly without changing the basic qualitative results). Denoting the free-electron density by n, we find the following system of kinetic equations for the trapping:

$$\dot{N}_+ = c_1(-nN_+ + n_1N_0\varphi_1) + c_3(N_0^2 - n_3N_-N_+), \tag{1}$$

$$\dot{N}_- = c_2(nN_0 - n_2N_-\varphi_2) + c_3(N_0^2 - n_3N_-N_+). \tag{2}$$

Here c_1 and c_2 are the coefficients of the trapping of conduction electrons to D_+ and D_0 centers, and c_3 is the coefficient of the "trapping" of an electron from one D_0 level to another. For obvious reasons, we have $c_1 > c_2$, while the difference $c_2 - c_3$ may be of either sign.

The functions φ_1 and φ_2, which depend on the electric field E, describe the change in the probability for the return of electrons into the conduction band under the influence of the electric field E. The analogous function $\varphi(E)$ also determines the field dependence of the mobility which results from the trapping of charge carriers by shallow pockets, followed by a rapid return to the band (these traps should not be identified with the D_0, D_-, or D_+ centers).

We will be interested in the region of strong fields and in the case $\varphi_1 = \varphi_2 = \varphi \gg 1$, $d\ln\varphi/d\ln E \gg 1$. If, in particular, we use the expression

$$\varphi = \sqrt{E_0/E}\,\sinh\sqrt{E/E_0}, \qquad E_0 = (kT)^2\epsilon/4e^3, \tag{3}$$

where ϵ is a quantity near the high-frequency dielectric constant, then we find $d\ln\varphi/d\ln E = \frac{1}{2}(E/E_0)^{1/2}$. The quantities n_1, n_2, and n_3 are given by the following expressions (we place the origin of the energy scale at E_+; we then have $E_- = 2E_0 - U$)

$$n_1 = g_1N_c\exp\left(\frac{E_0 - E_c}{kT}\right) + \frac{J_1\sigma_1}{c_1}; \qquad n_2 = g_2N_c\exp\left(-\frac{U}{kT} + \frac{E_0 - E_c}{kT}\right)$$
$$+ \frac{J_2\sigma_2}{c_2}; \qquad n_3 = \exp\left(-\frac{U}{kT}\right) + \frac{J_3\sigma_3}{c_3}. \tag{4}$$

Here the g_i are the ordinary statistical weights ($i = 1, 2$), and J_i and σ_i are the flux density of photons of the corresponding frequency and the cross section for the capture of a photon by the centers $D_-(i = 2)$ and $D_0(i = 1)$ for the transition $D_0 \to D_+$ or $i = 3$ for the transition $2D_0 \to D_+ + D_-$).

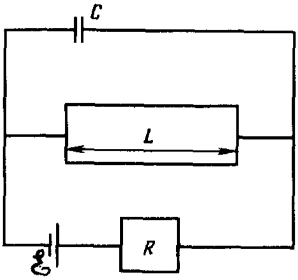

FIG. 1.

The equation for the electric field in the sample is the same as in Ref. 1:

$$\dot{E} = \frac{4\pi}{\epsilon^*}\left[\frac{\mathscr{E}}{SR} - \left(\sigma + \frac{L}{SR}\right)E\right], \tag{5}$$

where $\sigma = en\mu_0\varphi(E)$, $\mu_0 = \mu(E+0)$, $\epsilon^* = \epsilon = 4\pi CL/S$, S is the cross-sectional area of the sample, and the rest of the notation is explained in Fig. 1.

Equations (1), (2), and (5) must be supplemented with the relations

$$N_0 + N_+ + N_- = N, \qquad n + N_- = N_+ \tag{6}$$

(the second of which is the condition of local neutrality). Relations (6), along with the condition $E \leqslant \mathscr{E}/L$, determine the "physical region," within which the initial values of the variables must be specified. According to (1), (2), and (5), the integral curves cannot go outside this region.

Dynamic system (1), (2), (5) is analyzed in the standard way. It is easy to see that in the physical region there may be no more than two singularities, and in many cases of physical interest a singularity turns out to be unique. A sufficient condition for this situation is the satisfaction of the inequalities $n_1 N_D^{-1}\varphi > 1/2$ and $n_2 n_1^{-1} < 4$(if $c_2 \gg c_3$) or $n_e N_D^{-1} < 4$(if $c_2 \ll c_3$).

The conditions for the stability of the singularities are rather lengthy, but the (overly stringent) sufficient condition can be written in the simple form

$$\tau_m^{-1}\frac{d\ln\varphi}{d\ln E} < \frac{n_1 n_2 c_1 c_2}{n_1 c_1 + n_2 c_2}, \qquad c_2 \gg c_3, \tag{7a}$$

$$\tau_m^{-1}\frac{d\ln\varphi}{d\ln E} < \frac{n_1 n_3 c_1 c_3}{n_1 c_1 + n_3 c_3}, \qquad c_2 \ll c_3. \tag{7b}$$

Here $\tau_M^{-1} = 4\pi e\mu_0 n/\epsilon^*$ is the reciprocal of the Maxwellian relaxation time in the material, which has a conductivity $e\mu_0 n$ and a dielectric constant ϵ^*.

The fact that the derivative of the function φ appears on the left sides of inequalities (7a) and (7b) should not be surprising. This derivative arises from the field dependence of the mobility. If the functions describing the field dependence of the mobility and of the electron-production probabilities were different ($\varphi_1 \neq \varphi$, $\varphi_2 \neq \varphi$) the ratios $\varphi_1\varphi^{-1}$ and $\varphi_2\varphi^{-1}$ would have appeared on the right sides of (7a) and (7b).

It is easy to show that under conditions of an instability the singularity will be a saddle point, and one of the roots of the corresponding secular equation will be real and negative, while the two others will be complex with a positive real part. The latter turns out to be smaller in absolute value than the negative real root; i.e., we have the conditions of the Shil'nikov theorem[4]: a countable set of saddle-type periodic motions arises near a singularity. According to Ref. 5, this assertion means that a homoclinic structure must arise here. In other words, stochastic self-excited oscillations of the electric field and of the densities of free and bound charges in the sample arise in a system of this type (there are consequently oscillations in the current in the circuit and/or the voltage across the sample and across the load resistance).

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