Supersymmetry and anomalous dimensionalities of quasiparton operators in quantum chromodynamics

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A class of quasiparton operators of arbitrary twist, which is closed with respect to the evolution equations, is introduced. For this class of operators, the evolution equations are Faddeev equations with a binary interaction. The role of the interaction is played by a matrix of anomalous dimensionalities of operators of twist 2. The subgroup of supersymmetry transformations is used to derive relations between the elements of the anomalous-dimensionality matrix in quantum chromodynamics.

The cross sections for deep inelastic processes of the *ep*-scattering type can[1,2] be expressed in field theory in terms of the distributions $n_h^j(x)$ of the number of partons, j (quarks and gluons with different helicities), in the fraction (x) of the energy in the system in which the hadron h has an infinite momentum. The evolution equations of the parton distributions[1,2] yield the functional dependence of n_h^j on the momentum transfer Q.

In general, a description of the amplitudes for deep inelastic processes would require the introduction of more-general quantities: the so-called parton correlation functions, consisting of products of the wave functions of the initial and final hadrons, with generally different numbers of partons.[3,4] In an operator expansion, the amplitudes are represented as the sums of factorized expressions in which one factor is a matrix element of some local operator Q_r in a hadron state, while the second factor is the weight of the corresponding operator in the expansion of the original operators (of the nature of electromagnetic currents). The Q dependence of the amplitudes is determined completely by the dependence of the matrix elements of the operator O_r on the normalization point μ.

The operators O_r are tensors constructed from the fields ψ, $\bar{\psi}$, and $G_{\mu\nu}$ and the covariant derivatives D_μ in accordance with a definite Young tableau. By virtue of the Lorentz invariance of the matrix of anomalous dimensionalities, γ_{rs} for these operators does not depend on their projections. It is convenient to choose the component of the tensor which is found by turning the tensor with respect to those indices that are used to perform the symmetrization and to single out the traces, with the light-like vector n_μ:

$$n_\mu = \frac{q'}{pq}\,, \qquad q' = q - \frac{q^2}{pq}p\,, \qquad q'^{\,2} = 0, \tag{1}$$

where q is the momentum of the virtual photon, and p is the momentum of the proton (in *ep* scattering). To denote this convolution, we will replace the corresponding tensor indices by dots. In addition, we choose a light-like gauge for the gluon field: $A_\bullet = 0$.

We introduce a class of composite operators that satisfy two conditions: 1) These operators do not have any explicit dependence on the interaction constant g. 2) In a calculation of the matrix elements of these operators in parton states, we may assume that the partons lie on the mass shell. We call these new entities "quasiparton operators." It can be shown that these operators can be constructed only from the following structures: $\gamma_\bullet \psi$, $\bar{\psi}\gamma_\bullet$, $\gamma_\sigma^\perp$, $A_\sigma^\perp$, and $\partial_\bullet$, where $\perp$ means the projection of the vector onto the space orthogonal to the (p, q) plane. The quasiparton operators are definite components of gauge-invariant tensors which have the minimum possible twist n for a given number of fields n. All operators of twist 3 which describe deep inelastic scattering in the polarized case[3] and also most operators of higher twist[4] belong to the class of quasiparton operators.

An important property of quasiparton operators is that they do not mix with other operators upon renormalizations. It can be shown that in the single-loop approximation the evolution equations for these operators are the Faddeev equations for a system of n particles with a binary interaction (cf. Ref. 3), whose role is played by a matrix of anomalous dimensionalities of operators of twist 2 (which are generally colored). Making use of the uncolored nature of the quasiparton operators, we can define this matrix in such a way that all its elements are infrared-finite. This matrix can also be partially diagonalized by making use of conformal invariance.[5] The corresponding operators are

$$P^i(j) = \frac{(j\,!)^2(j+1)!\,(j-1)!}{2(2j)!} \sum_{k=0}^{j-2} \frac{A_\rho\,(-i\overleftarrow{\partial}_\bullet)^k\, O^i\,(i\,\partial_\bullet)^{j-k-1} A_\sigma}{k\,!\,(k+2)\,!\,(j-k)!\,(j-k-2)!}\,, \tag{2}$$

$$Q^i(j) = \frac{(j\,!)^2(j+2)!\,(j-2)\,!}{(2j)\,!} \sum_{k=0}^{j-1} \frac{\bar{\psi}\,(-i\overrightarrow{\partial}_\bullet)^k\,\Gamma^i\,(i\,\partial_\bullet)^{j-k-1}\,\psi}{k\,!\,(k+1)!\,(j-k)\,!\,(j-k-1)\,!}\,, \tag{3}$$

$$R^i(j) = \frac{j\,!\,(j-1)\,!\,(j+1)!\,(j+2)!}{(2j+1)\,!} \sum_{k=0}^{j-1} \frac{\bar{\psi}\,(-i\overleftarrow{\partial}_\bullet)^k\gamma_\bullet\,\gamma_\rho\,O^i(i\,\partial_\bullet)^{j-k} A_\sigma}{k\,!\,(k+1)!\,(j-k+1)\,!\,(j-k-1)!}\,, \tag{4}$$

where $i = S, P, T$ correspond (by convention) to scalar, pseudoscalar, and tensor variants:

$$\begin{gathered} O^S = -\,\delta^\perp_{\rho\sigma}\,, \quad O^P = -\,i\epsilon^\perp_{\rho\sigma}\,, \quad O^T = \frac{1}{2}(\delta^\perp_{\mu\rho}\,\delta^\perp_{\nu\sigma} + \delta^\perp_{\nu\rho}\,\delta^\perp_{\mu\sigma} - \delta^\perp_{\mu\nu}\,\delta^\perp_{\rho\sigma}), \\ \Gamma^S = \gamma_\bullet\,, \qquad \Gamma^P = \gamma_\bullet\,\gamma_5\,, \quad \Gamma^T = \gamma_\bullet\,\gamma^\perp_\mu\,. \end{gathered} \tag{5}$$

The matrices O^i and Γ^i also contain a color structure, which is not indicated here. As examples, we list some previously unknown anomalous dimensionalities for the spinor operators $R^i(j)$ in (4):

$$\gamma_R^S(j) = \gamma_R^P(j) = -(NP_3 + P_6 - P_{15})\left(S_{j-1} + S_{j+2} - \frac{5}{3} + \frac{n_f}{6N}\right)$$
$$-\left(\frac{1}{N}P_3 + P_6 - P_{15}\right)\frac{2(-1)^j}{j(j+1)(j+2)} \quad ; \tag{6}$$
$$\gamma_R^T(j) = -(NP_3 + P_6 - P_{15})\left(S_j + S_{j+1} - \frac{5}{3} + \frac{n_f}{6N}\right) + \left(\frac{1}{N}P_3 + P_6 - P_{15}\right)\frac{(-1)^j}{j+1} \quad .$$

Here $S_j = \Sigma_{k=1}^{j} 1/k$; P_3, P_6, and P_{15} are operators which project onto definite color states of the quark-gluon system [the subscript specifies the dimensionality of the corresponding multiplet for the SU(3) group]; N is the number of colors; and n_f is the number of flavors. The operators, being total derivatives of expressions (2)–(4) of the type $\partial_{\bullet}^{n} P$, have the same anomalous dimensionalities by virtue of translational invariance. Knowing the anomalous dimensionalities of operators (2)–(4), we can write evolution equations for quasiparton operators of arbitrary twist.

The elements of the matrix of anomalous dimensionalities are not independent but are instead related by several relations. As an example, we consider the Yang-Mills supersymmetry theory, in which A and ψ transform in accordance with an adjoint SU(N)–group representation, and ψ is a Majorana field. This theory is invariant under the supersymmetry-transformation subgroup:

$$\delta A_\nu^\perp = \frac{1}{2}\bar{\psi} A_\nu^\perp \gamma_{\bullet} \eta = -\frac{1}{2}\bar{\eta}\gamma_{\bullet}\gamma_\nu^\perp \psi ;$$
$$\delta \psi = -i\partial_{\bullet}\hat{A}_\perp \eta ; \quad \delta\bar{\psi} = -i\bar{\eta}\,\partial_{\bullet}\hat{A}_\perp , \tag{7}$$

where the anticommuting parameters η are of the form $\eta = \hat{p}\xi$. Transformations (7) do not remove the composite operators from the class of quasiparton operators. Applying (7) to operators (2)–(4), we can construct irreducible supermultiplets from them. In particular, for colorless operators these supermultiplets are

$$O^S(j) \pm \frac{i}{2}\partial_{\bullet}\tilde{O}^P(j-1), \quad (1 \mp \gamma_5)R^S(j), \qquad \text{even } j , \tag{8}$$

$$O^P(j) \pm \frac{i}{2}\partial_{\bullet}\tilde{O}^S(j-1), \quad (1 \mp \gamma_5)R^S(j), \qquad \text{odd } j , \tag{9}$$

$$\left.\begin{array}{ll} P^T(j), & R^T(j+1) - \dfrac{j+2}{2j+1}\dfrac{i}{2}\partial_{\bullet}R^T(j) \quad (10) \\ Q^T(j), & R^T(j+1) - \dfrac{j-1}{2j+1}\dfrac{i}{2}\partial_{\bullet}R^T(j) \quad (11) \end{array}\right\} \quad \text{even } j ,$$

where

$$O^i(j) = Q^i(j) + P^i(j), \quad \tilde{O}^i(j) = -\frac{j+2}{2j+1}Q^i(j) + \frac{j-1}{2j+1}P^i(j) . \tag{12}$$

All the terms of each of the supermultiplets (8)–(11) have the same anomalous dimensionality. Consequently, for the operators $P^{S,P}(j)$ and $Q^{S,P}(j)$, for example, the eight elements of the matrix of anomalous dimensionalities in the supersymmetry theory can be expressed in terms of two parameters. Furthermore, multiplets (10) and (11) have an identical anomalous dimensionality, since $R^T(j+1)$ and $R^T(j)$ cannot mix upon renormalization. The transformation to quantum chromodynamics is made by multiplying the matrix elements by the ratio of the operators projecting onto colorless states for cases in which the quarks transform under the fundamental and adjoint representations. One of the relations that follows from this transformation was discovered empirically in Ref. 6.

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