# Theory of the fast evolution of the magnetic field in a two-component electron plasma

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In a plasma containing a small group of hot electrons, the generation of a magnetic field by virtue of the thermal emf rapidly goes into a nonlinear dynamic process that leads to a "flipping" of the magnetic-field profile.

In this letter we consider a modification of the generation of a magnetic field by virtue of the thermal emf (see Ref. 1, for example) in a plasma with two electron temperatures. In practice, a situation of this type may arise in the interaction of an electron beam with an anode foil. Knowing the magnetic field in this case is very important: The efficiency at which the beam energy is absorbed is determined by the field, as was shown by Rudakov.[2] Furthermore, as we will see below, in two-component electron hydrodynamics it is possible to qualitatively incorporate some effects that stem from "infinite-dimensional" hydrodynamics: a kinetics which substantially changes the purely hydrodynamic picture. Aliev *et al.*[3] have called attention to the nontrivial nature of the two-component description.

We are thus interested in the fast dynamics of a magnetic field in a plasma containing a small admixture of hot electrons: $n_h + n_c = zn_i$, where $n_h \ll n_c$, and $T_c = 0$. The ion density can be assumed given in this problem, since the massive ions cannot move over the typical "electron" times. We also assume $\omega_{He}\tau_e \gg 1, \beta = 8\pi p/H^2 \gg 1$ (with the idea of also applying the results to a collisionless plasma, and since there is no substantial change in the field generation in single-fluid hydrodynamics), and $l \gg c/\omega_{pe}$. (Here $l$ is a scale length of the problem. In this case we can ignore the electron inertia. Strictly speaking, in order to ignore the inertia of the beam electrons, we also need to satisfy the inequality $mv_h^2/2 \ll T$, which holds at $l \gg v_{Te}/\omega_{He}$. See Ref. 4 for the opposite case.) After these comments, the system of equations becomes

$$0 = -n_c e\mathbf{E} - \frac{e}{c} n_c [\mathbf{v}_c \mathbf{H}];$$

$$0 = -\frac{e}{c} n_h [\mathbf{v}_h \mathbf{H}] - T\nabla n_h; \qquad (1)$$

$$n_c \simeq zn_i(\mathbf{r});$$

$$\frac{\partial n_h}{\partial t} + \operatorname{div} n_h \mathbf{v}_h = 0;$$

  

$$\frac{\partial \mathbf{H}}{\partial t} = - c \,\mathrm{rot}\, \mathbf{E} ;$$

$$\mathrm{rot}\, \mathbf{H} = - \frac{4\pi}{c} e(n_c \mathbf{v}_c + n_h \mathbf{v}_h).$$

In the second equation we have omitted a term with the electron field, since it contains the small parameter $n_h/n_c$. Making use of the small quantities $\beta^{-1}$ and $n_h/n_c$, we can reduce system (1) to two equations for $n_h$ and $\mathbf{H}$:

$$\frac{\partial n_h}{\partial t} = \frac{cT}{e} \,\mathrm{div}\, \frac{1}{H^2} [\mathbf{H}, \nabla n_h ] ;$$

$$\frac{\partial \mathbf{H}}{\partial t} = \frac{cT}{e} \,\mathrm{rot}\, \frac{\nabla n_h}{n_c} .$$

We consider two cases. If $\mathbf{H}\|\mathbf{e}_z$, and the electron flow occurs in the $(r, \varphi)$ plane, then we can write

$$\frac{\partial n_h}{\partial t} = \frac{cT}{er} \frac{\partial(n_h, 1/H)}{\partial(r, \varphi)} ; \tag{2}$$

$$\frac{\partial H}{\partial t} = \frac{cT}{er} \frac{\partial(1/n_c, n_h)}{\partial(r, \varphi)} . \tag{3}$$

We see that under otherwise equal conditions the evolution of the density of hot electrons occurs a factor of $n_c/h_h$ more rapidly than the evolution of the magnetic field. Since $n_h$ and $H$ should vary at identical rates over time, this rapid evolution leads to the establishment of an $h_h(H)$ dependence, i.e., to a mutual cancelling of the two large terms on the right side of (2). We will demonstrate this situation in a simple example. We assume that at $t = 0$ we have $n_h = n_h(\varphi)$ and $H = H(r)$. It then follows from (2) that $n_h = n_h(\varphi - f(r)t)$, where $f(r) = cT/(er)\partial H^{-1}/\partial r$ (within terms on the order of $n_h/n_c$, $H$ is independent of the time). In general, we would have $f(r) \neq \mathrm{const}$, and the evolution of the hot-electron density would be accompanied by a "twisting" of the contour lines of $n_h$, and after a few "revolutions" we would have a strong $n_h(r)$ dependence and thus a strong $n_h(H)$ dependence. A general case can be reduced to the simple case through a transformation to a curvilinear coordinate system in the $(r, \varphi)$ plane, with axes directed parallel and perpendicular to the lines of $\mathbf{H} = \mathrm{const}$. It is easy to see that an analogous effect occurs in the pure kinetics for hot electrons: If the effect of the electric field is ignored, the drift of particles along the $H = \mathrm{const}$ lines gives rise to a dependence $p_h(H)$ because of the difference in velocities on different lines (a similar effect was found in the one-dimensional case in Ref. 5). After this functional dependence is established, the magnetic-field generation described by Eq. (3) gives way to a nonlinear dynamics, which generally involves a "breaking" of the $H$ profile. After this event, either the electron inertia[6] (in the collisionless case) or the collisions of electrons with ions[4,7] comes into play. In a plane geometry, this "filamentation" is a purely drift effect.

In the second case, with $\mathbf{H}\|\mathbf{e}_\varphi$, and with the electron flow along $r$, $z$, we must (in the collisionless case) take into account a possible difference between the temperatures along and across the magnetic field. In a quasihydrodynamic description, we must now use the following equation for the current of hot electrons[8]:

$$n_h \mathbf{v}_h = \frac{cn_h}{eH^2}\left(\frac{T_\perp}{H}[\nabla H, \mathbf{H}] + \frac{T_\parallel}{r^2}[\mathbf{H}, \mathbf{r}]\right) + \frac{cT_\perp}{e}\operatorname{rot}\frac{n_h \mathbf{H}}{H^2}.$$

In place of (2) and (3) we have

$$\frac{\partial n_h}{\partial t} = \frac{cT_\perp}{er}\frac{\partial(r^{1-\alpha}H^{-1}, n_h r^\alpha)}{\partial(r, z)}; \tag{4}$$

$$\frac{\partial H}{\partial t} = \frac{cT_\perp}{e}\frac{\partial(r^\alpha n_h, r^{-\alpha}n_c^{-1})}{\partial(r, z)}. \tag{5}$$

Here $\alpha = (T_\perp - T_\parallel)/T_\perp$, and the temperatures, as before, are assumed constant. We see that, again in this case, after the rapid establishment of the functional relationship between $n_h r^\alpha$ and $r^{\alpha-1}H$, the magnetic-field generation again goes into a nonlinear dynamics which ultimately causes a "filamentation" of **H** due to a breaking, but this dynamics (provided that $T_\parallel \neq 0$) is no longer a purely drift dynamics, and in the case of radial motion a magnetic field can arise.

In summary, when an electron fluid consists of two components, there can be important changes in the picture of the field generation. An effect similar to that discussed here was noted in a numerical simulation by Yabe *et al.*,[9] but the analytic derivation of the effect that they offered was based on an incorrect use of the parameter $n_h/n_c$. We note in conclusion that discarding the assumption $T_c = 0$ will not affect any of the results derived above aside from leading to a change in notation, $T \to T_h$, in Eqs. (2)–(5), even if $n_c T_c \gtrsim n_h T_h$.


[1]A. A. Bol'shov, Yu. A. Dreĭzin, and A. M. Dykhne, Pis'ma Zh. Eksp. Teor. Fiz. **19**, 288 (1974) [JETP Lett. **19**, 548 (1974)].
[2]L. I. Rudakov, Fiz. Plazmy **4**, 72 (1978) [Sov. J. Plasma Phys. **4**, 40 (1978)].
[3]Yu. M. Aliev, V. Yu. Bychenkov, and A. A. Frolov, Fiz. Plazmy **8**, 125 (1982) [Sov. J. Plasma Phys. **8**, 71 (1982)].
[4]A. S. Kingsep, L. I. Rudakov, and K. V. Chukbar, Dokl. Akad. Nauk SSSR **262**, 1131 (1982) [Sov. Phys. Dokl. **27**, 648 (1982)].
[5]A. V. Gordeev, L. I. Rudakov, and V. Yu. Shuvaev, Zh. Eksp. Teor. Fiz. **85**, 155 (1983) [Sov. Phys. JETP **58**, 91 (1983)].
[6]A. A. Chernov and V. V. Yan'kov, Fiz. Plazmy **8**, 931 (1982) [Sov. J. Plasma Phys. **8**, 522 (1982)].
[7]Yu. V. Mokhov, A. S. Kingsep, and K. V. Chukbar, Fiz. Plazmy **10**, 854 (1984) [Sov. J. Plasma Phys. **10**, 495 (1984)].
[8]T. F. Volkov, in: Reviews of Plasma Physics, Vol. 1 (ed. M. A. Leontovich), Plenum, New York (1965).
[9]T. Yabe, K. Mima, T. Sugiyama, and K. Yoshikawa, Phys. Rev. Lett. **48**, 242 (1982).



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