Dynamics of an isolated Bloch line

A. K. Zvezdin and A. F. Popkov

(Submitted 6 November 1984)
Pis'ma Zh. Eksp. Teor. Fiz. **41**, No. 3, 90–92 (10 February 1985)

The nonlinear dynamics of a vertical Bloch line in a thin film is analyzed. The velocity of the line is found as a function of the external force. Approximate equations that are derived give a concise description of a vertical Bloch line as a material particle of definite mass and mobility. The limiting velocities of the lines are studied.

The dynamic properties of vertical Bloch lines have recently been the subject of active experimental and theoretical research.[1–3] In the present letter we report what is apparently the first attempt to analyze the nonlinear dynamics of vertical Bloch lines, their limiting velocities, etc.

1. We consider a material with a large uniaxial anisotropy ($Q = K/2\pi M^2 \gg 1$, where K is the uniaxial-anisotropy constant, and M is the saturation magnetization). We start from the Slonczewski equations,[1]

$$(2M\alpha/\Delta\gamma)\dot{q} - \sigma\vec{\nabla}^2 q + 2MH'q = 2M(H_z - \gamma^{-1}\dot{\Psi}), \tag{1a}$$

$$\gamma^{-1}\Delta^{-1}\dot{q} = 2\pi M \sin 2\Psi - 2AM^{-1}\vec{\nabla}^2\Psi + (\pi H_x/2)\sin\Psi$$
$$- (\pi H_y/2)\cos\Psi + \alpha\gamma^{-1}\dot{\Psi}, \tag{1b}$$

$$(\partial q/\partial z)\Big|_{z=\pm h/2} = (\partial\Psi/\partial z)\Big|_{z=\pm h/2} = 0, \tag{1c}$$

where q is the coordinate of the center of the domain wall, $H_y(z) = -H_y(-z)$ is the demagnetizing field defined in Ref. 1, $H = (H_x, 0, H_z)$ is the external magnetic field, α is the dimensionless decay constant, and γ is the gyromagnetic ratio. The Z axis is along the axis of easy magnetization, and the plane of the domain wall is the XZ plane. The azimuthal angle Ψ determines the departure of **M** from the XZ plane; $\Delta = (A/K)^{1/2}$; $\sigma = 4(AK)^{1/2}$; H' is the gradient of the magnetic field which holds the domain wall in the XZ plane; and h is the thickness of the magnetic film. To see the basic features of the dynamics of the vertical Bloch lines we consider the case of thin films, $h \lesssim \Delta_L$, where $\Delta_L = (A/2\pi M^2)^{1/2}$ is the thickness of the vertical Block line when we can ignore the "twisting" of a domain wall.[4] To suppress the bending instability of domain walls in thin films, we need to satisfy the condition[5] $H' > 4\pi^2 M h^{-1} \cdot \exp(-\sigma/2M^2 h)$. We will consider the cases of thick films and plates separately. In the static case, with $\dot{q} = \dot{\Psi} = H_x = 0$, system (1) has the known solutions

$$q = \text{const}, \quad \tan(\Psi/2) = \pm \exp[(x - x_0)/\Delta_L], \tag{2}$$

where x_0 is the coordinate of the center of the Bloch line, and the plus and minus signs correspond to different directions of the spin rotation in the line. This problem has two scale lengths, Δ_L and $l = (\sigma/2MH'\Delta)^{1/2}$, which determines the size of the perturbed region of the wall. This problem can be solved analytically only in two asymptotic cases, $\Delta_L l^{-1} \gg 1$ and $\Delta_L l^{-1} \ll 1$, i.e., for large and small gradients of the field H'.

2. We first consider the case $\Delta_L l^{-1} \gg 1$. Ignoring the first and second terms on the left side of Eq. (1a) in comparison with the third, and substituting the expression for q into Eq. (1b), we find

$$\ddot{\Psi} - c^2 \nabla^2 \Psi + \omega_g^2 \sin\Psi \cos\Psi = \gamma \dot{H}_z - \alpha\omega_1 \dot{\Psi} - \Omega_x \omega_1 \sin\Psi + \Omega_y \omega_1 \cos\Psi, \tag{3}$$

where

$$\omega_g = \gamma(4\pi M H'\Delta)^{1/2}, \quad \Omega_{x,y} = \pi\gamma H_{x,y}/2, \; c = \Delta_L \omega_g, \text{ and } \omega_1 = \gamma H'\Delta.$$

If $\dot{H}_z, H_y = 0$, this equation has the exact solution[6]

$$\tan(\Psi/2) = \pm \exp[(x - \dot{x}_0 t)/\Lambda], \tag{4}$$

where $\Lambda = \Delta_L[1 - (\dot{x}_0/c)^2]^{1/2}$, $\dot{x}_0 = \mu_L H_x[1 + (\mu_L H_x/c)]^{-1/2}$, and $\mu_L = (\pi/2)\gamma\Delta_L \alpha^{-1}$. Substitution of solution (4) into (1a) clearly shows that the first and second terms in (1a) are in fact smaller than the third if $\Delta_L l^{-1} \gg 1$ and $\alpha < 1$, justifying the assumptions made in the derivation of Eq. (3).

The limiting velocity of c is a bifurcation point of Eq. (3). When we go through this point on the (Ψ, Ψ') phase plane, the saddle-point singularities $(\Psi = 0, \Psi' = 0)$ and $(\Psi = \pi, \Psi' = 0)$ become nodes. We might note here that the limiting velocity c is equal to the phase velocity of magnons near walls in the linear part of their spectrum, $\omega = (\omega_g^2 + c^2 k^2)^{1/2}$, where k is the wave number. Another bifurcation arises at $\Omega_x \omega_1 = \omega_g (v < c)$, i.e., at $H_x = 4\pi M$, when the saddle point $(\Psi = \pi, \Psi' = 0)$ becomes an unstable node. We assume $H_x < 4\pi M$.

An exact solution of (4) can be constructed by perturbation theory, which gives an abbreviated description of a vertical Bloch line and makes it possible to study the time-varying dynamics of a vertical Bloch line and the effect on it of external perturbations, boundary and initial conditions, inhomogeneities of the medium, etc. Assuming $\Psi = \Psi_0 + \Psi_1$, and projecting the linearized version of Eq. (3) for Ψ_1 onto its eigenfunction $\partial_x \Psi_0$, which corresponds to a zero eigenvalue, we find the following equation for $x_0(t)$ from the condition for excluding secular terms[1)]:

$$\partial_t(m_L\dot{x}_0) + m_L\dot{x}_0/\tau_L = -\alpha m_L\mu_L\dot{H}_z + 2MH_x\pi\Delta, \tag{5}$$

where $m_L = m_0[1-(\dot{x}_0/c)]^{-1/2}$, $m_0 = 4MQ^{-1/2}\omega_1^{-1}\gamma^{-1}$, and $\tau_L^{-1} = \alpha\omega_1$. If we introduce the coercivity in Eq. (5) by the standard substitution $H_x \to H_x - H_L\,\mathrm{sgn}\dot{x}_0$, we can use this equation to describe the unidirectional "automotion"[1,2] of a vertical Bloch line under the influence of field pulses $H_z(t)$.

3. We now consider a weak gradient, $\Delta_L l^{-1} \ll 1$. Integrating Eq. (1a) under the condition $H_z = 0$ and under the assumptions $\dot{q} = -\dot{x}_0 q'$, $\dot{\Psi} = -\dot{x}_0\Psi'$, we find

$$q = \int G(x-y)\Psi'(y)dy\ , \tag{6}$$

where $G(x)$ is the Green's function of Eq. (1a). Substituting (6) into (1b), we find a nonlinear integro-differential equation for $\Psi(x)$. An equation for $x_0(t)$ can be found by the perturbation theory described above, by taking as the zeroth approximation the function (2), which is the solution of system (1) in the limit $\dot{x}_0 \to 0$. As a result, we find

$$\frac{m_L^*\ddot{x}_0}{\sqrt{1+(\dot{x}_0/V_0)^2}} + \frac{m_L^*\dot{x}_0}{\tau_L^*}\left[1 + \frac{\pi^2\Delta_L}{2\alpha^2 l}\,\frac{(\dot{x}_0/V_0)^2}{\sqrt{1+(\dot{x}_0/V_0)^2}}\right] = 2MH_x\pi\Delta\ , \tag{7}$$

where $m_L^* = 4\pi^3M^2\Delta l s^{-2}$, $\tau_L^* = \pi^3\gamma M\Delta_L l s^{-2}\alpha^{-1}$, $s^2 = 8\pi A\gamma^2$, and $V_0 = \gamma\Delta\alpha^{-1}(2H'\sigma/M)^{1/2}$. As the Bloch line moves, the domain wall becomes curved, and this curvature, $\delta q(x)$, creates an additional retardation of the moving Bloch line; this effect is ultimately responsible for the nonlinear functional dependence $\dot{x}_0(H_x)$ in Eq. (7).

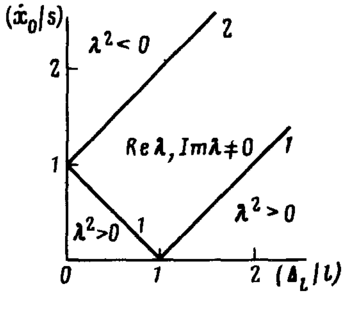

FIG. 1. Bifurcation diagram of system of equations (1). 1—The line $\dot{x}_0 = s|1-\Delta_L l^{-1}|$, which bounds the region in which Bloch lines exist ($\lambda^2>0$); 2—the line $\dot{x}_0 = s|1+\Delta_L l^{-1}|$, which separates the region of spin waves ($\lambda^2<0$) from the region of soliton solutions ($\mathrm{Im}\lambda$, $\mathrm{Re}\lambda \neq 0$).

4. In the general case, the limiting velocity is determined as a bifurcation point of system (1). Linearizing system (1) around the equilibrium position for $\alpha = H_x = H_y = 0$, and assuming $\delta q = q_0 \exp[\lambda (x - \dot{x}_0 t)]$, and $\delta \Psi = \Phi_0 \exp[\lambda (x - \dot{x}_0 t)]$, we find from (1)

$$\lambda^2 = \{1 + b^2 - (\dot{x}_0/s)^2 \pm [(1 + b^2 - (\dot{x}_0/s)^2)^2 - 4b^2]^{1/2}\}(2\Delta_L^2)^{-1}, \quad (8)$$

where $b = \Delta_L l^{-1}$. Figure 1 shows a bifurcation diagram of system (1). The limiting velocity $\dot{x}_0 = s|1 - \Delta_L l^{-1}|$ is determined from the vanishing of the discriminant in (8). The magnitude of this velocity, $s = \gamma(8\pi A)^{1/2}$, is the same as the critical velocity found for a domain wall by Enz.[7]

[1)]The demagnetizing field H_y, an odd function of z, is eliminated from the resulting equation (5) by the projection operation. The standard conditions for the applicability of the perturbation theory used here (see Ref. 8, for example) are determined by the inequalities $|\gamma \dot{H}_z|$, $(\dot{x}_0/\Delta_L)^2$, $|\ddot{x}_0/\Delta_L|$, $|\Omega_x \omega_1| \ll \omega_g^2$ for Eq. (5) and $|\dot{x}_0/s|$, $|\ddot{x}_0 l/s^2|$, $|\alpha \dot{x}_0/\gamma 4\pi M \Delta_L| \ll 1$ for Eq. (7).

[1]A. P. Malozernoff and J. C. Slonczewski, Magnetic Domain Walls in Bubble Materials. Applied Solid State Science, Supplement I (1979) (Russ. transl. Mir, Moscow, 1982).
[2]S. Konishi, IEEE Trans. Magn. **MAG-19**, 1838 (1983).
[3]V. I. Nikitenko, L. M. Dedukh, V. S. Gornakov, and Yu. I. Kabanov, Pis'ma Zh. Eksp. Teor. Fiz. **32**, 152 (1980) [JETP Lett. **32**, 140 (1980)].
[4]G. E. Khodenkov, Fiz. Met. Metalloved. **58**, 37 (1984).
[5]F. B. Hagedorn, J. Appl. Phys. **41**, 1161 (1970).
[6]A. K. Zvezdin, Pis'ma Zh. Eksp. Teor. Fiz. **29**, 605 (1979) [JETP Lett. **29**, 553 (1979)].
[7]U. Enz. Helv. Phys. Acta **37**, 245 (1964).
[8]D. MacLaughlin and E. Scott, in: Solitons in Action (Russ. transl. Mir, Moscow, 1981).

Translated by Dave Parsons
Edited by S. J. Amoretty