Dramatic stimulation of tunneling by an rf field

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The correction to the argument of the tunneling exponential function which is linear in the amplitude of the field $\mathscr{E}\cos\Omega t$ is calculated for semiclassical potentials. In the frequency interval $V \gg \Omega \gg \omega$ (V is the potential amplitude, and ω is the oscillation frequency in the reversed potential) the effective field determining the barrier transmission is exponentially large in comparison with $\mathscr{E}$: $\mathscr{E}_{\rm eff} \sim \mathscr{E}\exp(\Omega\tau_s)$, where $\tau_s \sim \omega^{-1}$.

A semiclassical description of tunneling in terms of trajectories satisfying Newton's equation necessarily leads to the concept of a motion in imaginary time and thus lays the foundation for a method of complex classical trajectories.[1] In the present letter we wish to discuss the effects of motion in imaginary time as they would influence the tunneling of an alternating field $\mathscr{E}\cos\Omega t$ through a semiclassical potential barrier.

In imaginary time, $t = i\tau$, the field is of the form $\mathscr{E}\cosh\Omega\tau$, so that the field contribution to classically forbidden processes of the tunneling type is determined not by the field amplitude $\mathscr{E}$ but by the quantity $\mathscr{E}\cosh\Omega\tau_s$, where τ_s is the distance from the real axis to the singularity on the trajectory $x(t)$. For semiclassical potentials, the condition $V \gg \omega$ holds, where V is the potential amplitude, and ω is a scale frequency of the tunneling. We then have $\tau_s \sim \omega$, and over a broad frequency range $V \gg \Omega \gg \omega$ the effective amplitude of the alternating field, on which the transmission coefficient depends, is exponentially large in comparison with $\mathscr{E}$: $\mathscr{E}_{\rm eff} \sim \mathscr{E}_{\rm exp}\Omega\tau_s$. The quantity $\mathscr{E}_{\rm eff}$ in turn figures in the argument of the exponential function, so that this effect may be legitimately called a dramatic or giant effect. In this letter we offer a general formulation of the problem. The final result of these calculations is the correction, which is linear in $\mathscr{E}$, to the argument of the tunneling exponential function for potentials of a certain type.

We know that the wave functions can be sought with exponential accuracy in the form $\psi(x, t) = \exp[iS(x, t)]$, where $S(x, t)$ is the classical action, and x and t lie on the classical trajectory of the particle, which is found from the equation

$$m d^2x/dt^2 = -dV/dx + \mathscr{E}\cos\Omega t. \tag{1}$$

We assume that a particle is incident on the barrier from the left. Since there is no classical trajectory connecting points with the real coordinates X_-, $t_-(X_- < 0)$ and X_+, $t_+(X_+ > 0)$, we consider the solutions of Eq. (1) on the contour C_+ in the plane of the complex variable t (Fig. 1). On the symmetric contour C_-, we have $x(t^*) = x^*(t)$. Far off to the right on $C_\pm$, the values of x and t are real, and $x > 0$. The solution of (1) depends on two arbitrary real parameters. Since we are assuming that the alternating field is turned off adiabatically as $t \to -\infty$, we choose one of these parameters in such a way that in the limit $t \to -\infty$ the total energy $E = m(dx/dt)^2/2 + V(x)$ would

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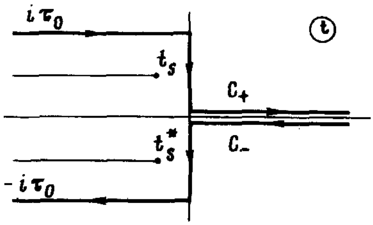

FIG. 1.

be given. Far off to the left in the t plane, the contours run parallel to the real axis, at distances $\pm i\tau_0$ from it. The quantity τ_0 is determined from the solution of Eq. (1) in such a manner that far off to the left on $C_\pm$ the coordinate x is real and negative. The second adjustable parameter specifies the time at which the motion begins: $x(t_- \pm i\tau_0) = X_-$.

The wave function on the part of the contour C_+ far off to the left, where we have $\mathscr{E}(t) = 0$, is related to the wave function $\psi(X_-, t_-)$, of real arguments, by

$$\psi(X_-, t_- + i\tau_0) = \psi(X_-, t_-)\exp(E\tau_0).$$

We thus find the tunneling probability to be

$$D = \exp(-A); \qquad A = -i \int_{C_+ + C_-} L\,dt,$$

where the Lagrangian has the same form as in Ref. 2:

$$L = \frac{m}{2}\left(\frac{dx}{dt}\right)^2 - V(x) + \mathscr{E} x \cos\Omega t + E.$$

In specific calculations it may be convenient to displace the integration path far to the left, where $\mathscr{E}(t) = 0$, taking into account the coupling of the contours to the singularities of the function $x(t)$.

We now calculate the correction to the argument of the tunneling exponential function which is the correction linear in the amplitude of the alternating field. Since the action is at an extremum, we find this correction to be

$$A_1 = -i\mathscr{E} \int_{C_+ + C_-} x(t)\cos(\Omega t)dt, \qquad (2)$$

whre $x(t)$ is the classical trajectory in the absence of an alternating field. For real $x(t)$, the change in t along the trajectory corresponds to the contour C_+ in Fig. 1. The tunneling between the turning points x_+ and x_- corresponds to the vertical part of the contour. The motion at $x > x_+$ corresponds to real time, while the motion to the left of x_- corresponds to a horizontal straight line separated from the real axis by τ_0, which is the tunneling time.

A trajectory of this shape corresponds to a potential $V(x)$ with a hump. The singularities of the analytic function $x(t)$ are related to those of the potential $V(x)$ in the complex x plane. We restrict the discussion of this point to barriers for which $V(x)$ has power-law singularities at certain points x_s and x_s^*, where it becomes infinite:

$$V(x) \simeq \kappa(x-x_s)^{\alpha}, \qquad x \to x_s.$$

Here $\alpha < 0$. The same is true of singularities of the type $V \simeq \kappa x^{\alpha}$ in the limit $x \to \infty$ with $\alpha > 0$. Near x_s we have

$$x(t) \simeq x_s + [-\kappa(2-\alpha)^2(t-t_s)^2/2m]^{1/(2-\alpha)},$$

where t_s is the complex time of the motion from x_+ to x_s:

$$t_s = (m/2)^{1/2} \int_{x_+}^{x_s} (E - V(x))^{-1/2}\, dx.$$

In order of magnitude, $\tau_s \equiv \operatorname{Im} t_s$ is the same as τ_0 and the reciprocal of the tunneling frequency ω. In the limit of high frequencies of the alternating field, $\Omega \gg \omega$, the integral in (2) is dominated by small regions of cuts near the singularities t_s and t_s^*. For the transmission coefficient, we ultimately find

$$D(\mathscr{E}) = D(0) \exp\left\{ \frac{2\pi\mathscr{E}}{\Omega} \exp(\Omega\tau_s) \left| \Gamma\left(\frac{2}{\alpha-2}\right)\right|^{-1} \left(\frac{|\kappa|(2-\alpha)^2}{2m\Omega^2}\right)^{1/(2-\alpha)} \right\}. \qquad (3)$$

The factor $\cos(\Omega t + \varphi_0)$ standing beside $\mathscr{E}$ (t is the time required for the particle to move to the barrier) is replaced by 1 or -1 in order to find the maximum transmission coefficient. In general, an average over the period corresponds to the replacement of the outer exponential function $\exp(z)$ in (3) by a Bessel function $I_0(z)$.

Our analysis is based on the assumption that the unperturbed problem contains a parameter ω, which is a measure of the frequency of the internal motion. For many-photon ionization of an atom,[3] in contrast, the quantity analogous to τ_s in (3) is itself determined by the amplitude of the alternating field.

In the particular case of the potential $V(x) = V/\cosh^2(x/a)$, we have $x_s = -i\pi a/2$, $\kappa = -Va^2$, $\alpha = -2$, and $\tau_s = \pi a(m/8E)^{1/2}$, and the expression (3) becomes

$$D(\mathscr{E}) = D(0) \exp\left[\frac{2\mathscr{E}}{\Omega}\left(\frac{\pi^2 V a^2}{2m\Omega^2}\right)^{1/4} \exp(\Omega\tau_s)\right]. \qquad (4)$$

The solution of a classical problem with an alternating field of finite amplitude will presumably give us a critical field value $\mathscr{E}_c(\Omega)$ at which the barrier transmission reaches a value on the order of unity. An important point is that at $\Omega \gg \omega$ the quantity $\mathscr{E}_c$ is exponentially small in comparison with the scale field at the barrier, $\sim \partial V/\partial x$.

An expression analogous to (4) also holds for the coefficient of the above-barrier reflection of a particle.

One example of a manifestation of this effect might be the acceleration of tunneling chemical reactions by an rf field. In this case, a typical value of the "semiclassicality parameter" is[4] $V/\omega \sim 10$. The typical frequencies of the alternating field under the condition $\Omega \lesssim V$ should then be $\Omega \sim 10^{15}$ s^{-1}.

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