Self-consistent equation for the magnetic mass of a gluon

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An equation is derived for the magnetic mass of a gluon with allowance for the entire series of nonperturbative diagrams for the polarization operator. The four-gluon vertex is eliminated in a self-consistent way. The calculation scheme is closed through the choice of a nonperturbative approximation of the three-gluon vertex. A $g^4T^2\ln(\mu/g^2T)$ behavior is predicted for the square magnetic mass of the gluon.

The magnetic mass of a gluon (the reciprocal infrared cutoff radius of the effective gluomagnetic interaction) is an extremely interesting topic for theoretical research in quantum chromodynamics. The study of this mass is directly related to the study of the infrared problem at $T\neq 0$ in the theory of non-Abelian gauge fields and requires a substantial departure from conventional perturbation theory. However, methods for nonperturbative calculations in quantum chromodynamics at $T\neq 0$ are still in a formative stage,[1,2] and no systematic algorithms are available. The construction of these algorithms is limited to a study of the simplest possibilities and is to a large extent heuristic. All that has been shown so far[3,4] is that the one-loop nonperturbative magnetic mass of a gluon is strictly zero, and that this result is gauge-independent, at least

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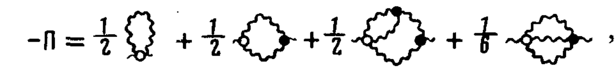
in the class of axial gauges. However, we do not know how this result will be modified when we take into account two-loop nonperturbative diagrams of the polarization operator, which, along with the one-loop diagrams, constitute the complete set of perturbation-theory diagrams. In this sense, the situation must be judged unsatisfactory, and the development of better methods for nonperturbative calculations is a problem of the greatest importance. An unresolved problem in these calculations results from the difficulty in constructing correct nonperturbative approximations for the vertex functions of the theory (both three-gluon and four-gluon vertices) and the complexity of the resulting expressions. On the other hand, there are several ways in which progress can be made. In particular, for certain nonperturbative-calculation schemes the four-gluon vertex can be eliminated in a self-consistent way, and the calculation scheme, which constitutes the complete set of diagrams of the perturbation theory, is closed through the use of only a nonperturbative approximation for the three-gluon vertex.

The nonperturbative calculations are carried out in the axial gauge, which is singled out for a non-Abelian theory of gauge fields by the simple form of the Slavnov-Taylor identities,

$$r_\mu \Gamma_3(r,p,q)^{abf}_{\mu\nu\gamma} = igf^{abf}\{D^{-1}_{\nu\gamma}(p) - D^{-1}_{\nu\gamma}(q)\},$$

$$r_\mu \Gamma_4(r,p,q,t)^{abcf}_{\mu\nu\gamma\lambda} = ig\,\{f^{acb}\,\Gamma_3(t,p,-t-p)^{fdb}_{\lambda\nu\gamma} + f^{adb}\Gamma_3(t,q,-t-q)^{fcb}_{\lambda\gamma\nu} + f^{afb}\,\Gamma_3(p,q,-p-q)^{dcb}_{\nu\gamma\lambda}\},\qquad(1)$$

and which, by virtue of the absence of propagating fictitious particles, allows a closure of the self-consistent calculations on nonperturbative diagrams of a single topological type. In quantum chromodynamics, the exact diagram representation for the polarization operator,



(2)

contains both single-loop and two-loop nonperturbative diagrams, each of which classes can be treated separately in any axial gauge. The simple schemes for nonperturbative calculations of the magnetic mass of the gluon in the gauge $A_4 = 0$ (Refs. 3 and 4) deal exclusively with single-loop nonperturbative diagrams and do not completely incorporate the last two diagrams in (2). The approximation of the three-gluon vertex in terms of the structure functions $G(k)$ and $F(k)$ of the exact gluon propagator,[1,5]

$$D_{ij}(k) = \frac{1}{k^2+G}\left(\delta_{ij} - \frac{k_i k_j}{\mathbf{k}^2}\right) + \frac{1}{k^2+F}\,\frac{k^2}{k_4^2}\,\frac{k_i k_j}{\mathbf{k}^2},\qquad(3)$$

is chosen to be the same as the exact asymptotic expression for $\Gamma_3(r,p,q)$:

$$\Gamma_3(0,-p,p)^{abc}_{eji} = igf^{abc}\,\frac{\partial D^{-1}_{ij}(p)}{\partial p_e}.\qquad(4)$$

It satisfies the first identity in (1). The result of these calculations,

$$m^2_{\text{mag}} = 0\,, \tag{5}$$

does not depend on the explicit form of the gauge in the class of axial gauges, and for the nonperturbative approximation being discussed here this result can be offered as rigorous.

Incorporating the two-loop nonperturbative diagrams of series (2) requires an approximation of a four-gluon vertex. The latter must be matched with identities (1) and constructed in terms of the functions $F(k)$ and $G(k)$ that determine (3). This approximation must also agree exactly with the first few orders of perturbation theory, and it must agree to the extent possible with the effective summation of all the other nonperturbative diagrams, which constitute the remainder for it. For the uniqueness of the approximation, these requirements are, of course, not sufficient, and the approximation of the four-gluon vertex that we are proposing here,

$$\Gamma_4(p,q,r,0)^{acdf}_{\mu\nu\gamma\rho} = \delta(p+q+r)\left\{(-igf^{dfb})\frac{\partial\Gamma_3(p,q,r)^{acb}_{\mu\nu\gamma}}{\partial r_\rho}\right.$$
$$\left. + (-igf^{cfb})\frac{\partial\Gamma_3(p,q,r)^{abd}_{\mu\nu\gamma}}{\partial q_\rho} + (-igf^{afb})\frac{\partial\Gamma_3(p,q,r)^{bcd}_{\mu\nu\gamma}}{\partial p_\rho}\right\}, \tag{6}$$

is the simplest nonperturbative realization of this approximation. For self-consistent calculations of the magnetic mass of the gluon, however, for which the problem becomes one of obtaining this value is the first nonvanishing approximation (or to prove that it is strictly zero), approximation (6) is useful for getting a comprehensive picture of the complexity and length of the calculations required.

The magnetic mass of the gluon is determined by the infrared limit ($p_4 = 0$, $|\mathbf{p}|\to 0$) of diagram series (2),

$$m^2_{\text{mag}} = G(0,0) = \frac{1}{2}\sum_i \Pi_{ii}(0,0)\,, \tag{7}$$

all of whose diagrams can now be summed completely, with the help of (6). The expression for the magnetic mass of the gluon,

$$m^2_{\text{mag}}\,\delta^{f,h} = \frac{1}{4}\sum_i \frac{1}{\beta^2}\sum_{p_4,q_4,r_4}(2\pi)^3\int\frac{d^3p}{(2\pi)^3}\frac{d^3q}{(2\pi)^3}\frac{d^3r}{(2\pi)^3}\,\delta^{(4)}$$
$$(p+q+r)\Gamma_4^{(0)}(0,r,q,p)^{fdcb}_{iksn} *$$
$$* D_{sg}(q)D_{kt}(r)(igf^{bah})\frac{\partial}{\partial p_i}\left[D_{nm}(p)\Gamma_3(-p,-q,-r)^{acd}_{mgt}\right], \tag{8}$$

results from simple algebraic manipulations and is fixed by the nonperturbative three-gluon vertex taken for arbitrary momenta. In the gauge $A_4 = 0$, this scheme for self-consistent calculations is of special interest, since the nonperturbative expression required for the three-gluon vertex has already been constructed,[4] and Eq. (8) can be solved by standard computational methods. Using the known expression for the seed four-gluon vertex (and summing over the group indices), we can simplify Eq. (8):

$$m^2_{\rm mag} = \frac{3g^4N^2}{4\beta^2} \sum_{p_4, q_4, r_4} (2\pi)^3 \int \frac{d^3p}{(2\pi)^3} \frac{d^3q}{(2\pi)^3} \frac{d^3r}{(2\pi)^3} \delta^{(4)}(p+q+r) \tag{9}$$

$$* \; D_{ns}(q) D_{it}(r) \frac{\partial}{\partial p_i} [D_{nm}(p) \Gamma_3(-p, -q, -r)_{mst}].$$

The expression in this form is the basic result of this study. In the derivation of (9), we used simple relations for the structure constants of the SU(N) groups, F^h_{ac} $(= if^{ahc})$,

$$\mathrm{Tr}(F^a F^b) = N\delta^{ab}, \quad \mathrm{Tr}(F^a F^b F^c) = \frac{N}{2} F^b_{ac}, \tag{10}$$

and a standard redefinition of the three-gluon vertex,

$$\Gamma_3(p, q, r)^{abc}_{mst} = (-igf^{abc}) \Gamma_3(p, q, r)_{mst}. \tag{11}$$

A solution in the form in (9) can be constructed by an iterative method by regularizing the infrared divergences of the integrals in (9) through a cutoff at momenta on the order of g^2T. This cutoff is a consequence of the assumption $m^2_{\rm mag} \neq 0$ and is self-consistent in the leading asymptotic behavior. The nonperturbative procedure for solving Eq. (9), which uses the exact propagators and vertex functions, does not require an additional infrared regularization, but it is extremely lengthy and goes beyond the scope of this letter. Here we can give the result of the zeroth iteration of Eq. (9), which is equivalent to a calculation of $m^2_{\rm mag}$ by standard perturbation theory of order g^4. Nonzero contributions to $m^2_{\rm mag}$ come from two two-loop diagrams of the zeroth iteration of the polarization operator (2) of order g^4:

$-m^2_{mag} = \frac{1}{12}$  $+ \frac{1}{4}$  . (12)

These diagrams can be calculated with the help of the seed propagators and the vertices

$$D^{(0)}_{ij}(k) = \frac{1}{k^2}\left(\delta_{ij} + \frac{k_i k_j}{k_4^2}\right), \tag{13}$$

$$\Gamma^{(0)}(-p, -q, -r)_{mst} = [\delta_{mt}(p-r)_s + \delta_{sm}(q-p)_t + \delta_{st}(r-q)_m].$$

The other two-loop diagrams of the standard perturbation theory do not contribute to m^2_{mag}, according to (5), and have thus been omitted from (12). Direct calculations of the diagrams require cumbersome algebraic transformations and imply the use of regularization of the $(1/k_4^2)^n$ singularities, which is characteristic of the $A_4 = 0$ axial gauge. After a regularization of these singularities, all the sums over k_4 are replaced by a single term of the sum with $k_4 = 0$, and the three-dimensional integrals that arise are evaluated by introducing an additional ultraviolet cutoff at momenta on the order of μ. The result of the calculations depends on g^2T in a nonanalytic manner:

$$m^2_{\text{mag}} = \chi^2 g^4 T^2 \ln(\mu/g^2T), \tag{14}$$

but the coefficient χ^2 cannot be determined unambiguously, since it depends on the choice of regularization parameters. A very important result, however, is $\chi^2 \neq 0$, so that the result in (14) differ from the results of Refs. 6 and 7, where the leading asymptotic behavior for m^2_{mag} was proportional to g^4T^2. The result found previously ($m^2_{\text{mag}} = 0$) in a nonperturbative single-loop calculation[3,4] is, of course, consistent with result (14) and is taken into account in the derivation of Eq. (9). The nonperturbative solution of Eq. (9) must confirm the result in (14) and allow a calculation of the coefficient χ^2.

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