Random self-modulation of two-dimensional structures on a liquid surface during parametric excitation

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Experiments show that the transition to turbulence at the parametrically excited surface of a heavy liquid occurs through the development of a spatial and temporal modulation of two-dimensional wavelike structures (lattices). When the pump exceeds a certain critical level, the periodic modulation regime gives way to a random regime. The transition to chaos involves an intermittence: a formation of dislocations.

1. The appearance of steady-state spatial structures is characteristic of the regimes preceding the onset of turbulence in many types of hydrodynamic flow. Examples are the Bénard rollers or cells in the case of thermal convection in a plane layer, Taylor vortices in Couette flow between rotating cylinders, and the annular vortices in submerged jets. As the deviation of a system from equilibrium increases, these structures—unit cells—will often begin to oscillate at a definite frequency and a finite amplitude. Examples are the azimuthal oscillations of Taylor vortices and the bending oscillations of the rollers in the case of thermal convection. As a result of the onset of these oscillating cells, the flow essentially becomes a chain or lattice of coupled elementary "generators." The nature of the transition from a regular lattice of oscillating cells (structures) to turbulence depends on the dimensions of the system. If the flow is occurring in a closed region that contains only a few cells, when the excitation exceeds the critical value, these cells will usually begin to break up and combine in an irregular way over time, with the result that a chaotic regime arises; i.e., in this situation, the cells are destroyed upon the onset of turbulence. If instead the dimensions of the system are much larger than those of the oscillator cells, there is the possibility in principle that the transition to chaos will take a completely different form: The cells will not break up but will simply go into a different type of oscillation, and the periodic modulation regime will give way to a stochastic regime. Indirect evidence suggests that this type of stochastic modulation of structures has been observed in a chain of Taylor vortices which arises in Couette flow between cylinders.[1]

In this letter we report the first observed transition to turbulence through a random modulation of two-dimensional structures on the surface of a viscous liquid in a periodically oscillating uniform gravitational field.

Regular structures excited parametrically on the surface of a liquid were, in fact, observed in the last century[2] (Faraday ripples), and they have now been studied in detail.[3] The transition to chaos, on the other hand, was first observed only very recently, and only in a resonator with a few cells.[4] It has been observed that in this case the transition to chaos results from the alternating disappearance of two different structures formed by different modes of the resonator.

2. In the present experiments, the transition to chaos is observed on a horizontal

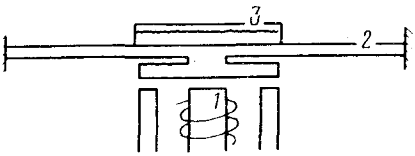

FIG. 1.

surface containing about 10^4 oscillator cells. Regardless of the shape of the boundaries, the turbulence arises against the background of two-dimensional structures, by virtue of a self-modulation of these structures which is random in time and space.

The experimental apparatus is shown in Fig. 1. A voltage of frequency $f = 112$ Hz, which corresponds to the fundamental resonance of a membrane (2) on which a cell (3) filled with the liquid (silicon oil) rests, is applied to an electromagnet (1). The acceleration amplitude R of the cell can be adjusted from 0 to 50 m/s^2. The density of the silicon oil is 0.6343 g/cm^3, its dynamic viscosity 2.807 cP, and its surface tension 67.04 erg/cm^2; all these measurements were taken at 20 °C. The thickness of the layer of oil in the cell is 0.5 cm, an the cell diameter is 30 cm. The liquid surface is photographed in reflected light, and then an optical spectrum analyzer is used to perform a spatial Fourier analysis of the image contrast.

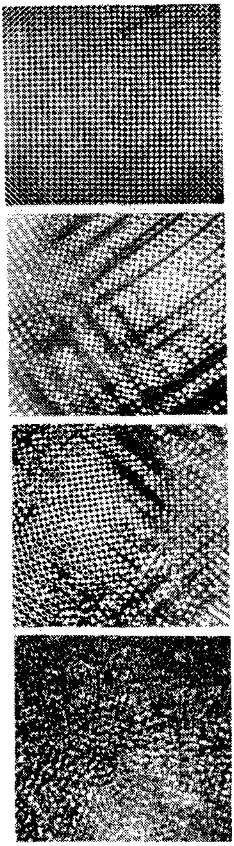

FIG. 2.

The following transitions are observed as the cell oscillation amplitude is increased. At a first critical acceleration amplitude $R_{cr1} = 6.3\ \text{m/s}^2$, a regular lattice with square cells arises at the surface of the liquid. This lattice is formed by two pairs of parametrically excited traveling waves (see the upper photograph in Fig. 2). The propagation directions of these pairs of waves are perpendicular, regardless of the shape of the cell boundaries.

A further increase in R gives rise to a modulation with a spatial period on the order of the dimensions of the cell, against the background of the tetrahedra of waves. The propagation directions of these waves are the same as the propagation direction of the parametrically excited waves that form the tetrahedra. With a further increase in R, the velocity of the modulation waves increases, and their spatial period decreases (Fig. 3). The transition to chaos occurs at $R_{cr2} = 12.5\ \text{m/s}^2$, where the shortest-wave modulation regime—the π-oscillation regime—becomes unstable. Immediately beyond the critical point, the modulation of the structures is exceedingly peculiar: We observe large blocks in which there is no disruption of the regularity in the arrangement of cells (laminar regions), bordered by small regions in which the arrangement of cells is chaotic (turbulence). As the extent to which the excitation exceeds the critical level, $r = (R - R_{cr2})/R_{cr2}$, is increased, these laminar regions shrink, while the turbulent regions grow. The behavior is illustrated in Fig. 2; the second photograph from the top corresponds to a preturbulence regime ($r = -0.2$); the next one down corresponds to a stochastic regime near the critical point ($r = 0.05$); finally, the lowest photograph shows developed chaos ($r = 1.4$). This picture of the transition from regular dynamics to stochastic dynamics is characteristic of the phenomenon of intermittence which has been studied in detail for low-dimensionality dynamic systems.[5]

Figure 4 shows some two-dimensional spatial Fourier spectra of the contrast corresponding to the regimes in Fig. 2. Just above the transition point, the spectrum still has some strong, discrete components; as r is increased further, these discrete components become weaker, eventually vanishing (the last spectrum in Fig. 4).

3. For a phenomenological description of the observed transition to turbulence, we can use a (spatially) discrete dynamic model. According to this model, the unit cells of the lattice should be nonlinear, parametrically excited oscillators that are coupled with each other. We wish to emphasize that a numerical study of a model of this type—a one-dimensional chain of coupled self-excited oscillators—has revealed[6] a

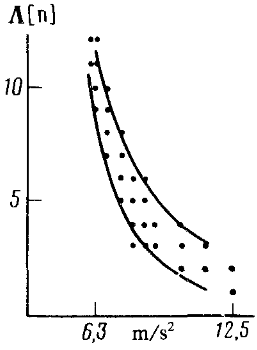

FIG. 3.

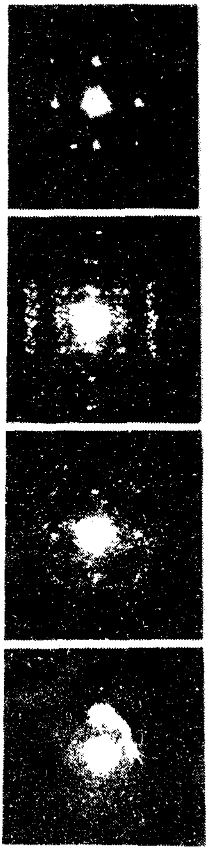

FIG. 4.

transition similar to that observed in the present experiments. As in these experiments, the onset of a stochastic behavior was preceded by a regular regime of a spatial and temporal self-modulation, which undergoes a transition at $r = 0$ through an intermittence to a regime of chaotic modulation.

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