The structure of a gluon string

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An effective approximate Lagrangian, which makes it possible to derive equations for average gluon fields, is constructed on the basis of quantum chromodynamics. The field distribution in a gluon string that connects a quark and an antiquark is found. The gluoelectric field directed along the string is surrounded by a circular gluomagnetic field. The fields are localized in a region whose transverse dimension is on the order of the confinement radius. The tension coefficient of a string is estimated.

The physical picture of vacuum fluctuations in quantum chromodynamics has recently begun to emerge. Studied particularly thoroughly have been the small-scale fluctuations, since the effective coupling constant is small on this scale and is given by a standard asymptotic-freedom formula[1]

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$$\frac{2\pi}{\alpha_s(\rho)} = b\ln(1/\Lambda\rho); \quad b = \frac{11N_c}{3}; \quad \Lambda = (50-200)\ \text{meV}. \tag{1}$$

Here ρ is the characteristic fluctuation scale, and N_c is the number of colors.

The nonperturbative fluctuations (instantons, for example) become important on the scale $\rho \sim R_c$ (R_c is the confinement radius). The value of α_s is still small in this case ($N_c\alpha_s \sim 0.6$–0.8). The nonperturbative effects lead to a sharp increase in $\alpha_s(\rho)$, so that a transition from perturbation theory to a strong-coupling regime occurs within a narrow scale range near R_c, as suggested by the recursive approach[2] and by numerical experiments.[3] The Coulomb interaction between quarks, expressed as a function of the distance R with $R = R_c (R_c^{-1} = 600$ meV), abruptly gives way to the law σR (σ is the tension coefficient of the string). The same conclusion can be reached by assuming that near R_c the instantons account for the nonperturbative contribution to $\alpha_s(\rho)$.[4]

Because the value of $\alpha_s(\rho)$ is small in the relevant scale range, we can write in the case of weak fields the effective action (the functional of the average field) in a local form

$$S_{eff}[A^a_\mu(x)] = -\frac{1}{16\pi\alpha_s(\rho)}\int d^4x\,[F^a_{\mu\nu}(x)]^2, \tag{2}$$

where ρ is the characteristic field distribution (far from the quarks, for example, it is the string radius and near a quark it is the distance from it), and $\alpha_s(\rho)$ is given by

$$\frac{1}{\alpha_s(\rho)} = \frac{\int d^4x d^4x'\,[E^a_n(x)E^a_n(x') + H^a_n(x)H^a_n(x')]\,\epsilon(x-x')}{\int d^4x\,[(E^a_n(x))^2 + (H^a_n(x))^2]}, \tag{3}$$

$$E^a_n = F^a_{0n}; \quad H^a_n = \frac{1}{2}l_{nmk}F^a_{mk};$$

here $F^a_{\mu\nu}$ is assumed to be an extremal value, i.e., it satisfies the equations of motion. This expression determines the unique relationship between α_s and $\epsilon(x)$, while $\epsilon(x)$ is linked to the polarization operator in a simple way.

In the presence of static quarks, we must add the term S_{int} to the effective action:

$$S_{\text{int}} = \int d^4x\, j_0^{(3)}(\mathbf{x},t)\,A_0^{(3)}(\mathbf{x},t), \tag{4}$$

$$j_0^{(3)}(\mathbf{x},t) = \frac{1}{2}[\delta(\mathbf{x}-\mathbf{x}_1) - \delta(\mathbf{x}-\mathbf{x}_2)]. \tag{5}$$

Here $\mathbf{x}_1$ and $\mathbf{x}_2$ are the quark and antiquark coordinates. This procedure can be used in gauges in which $A_0^{(1)}(\mathbf{x}_i) = A_0^{(2)}(\mathbf{x}_i) = 0$ at the points where the charges are located.

We restrict the discussion here to the gluodynamics, i.e., a theory that does not consider the quark fluctuations. Furthermore, we consider the case of weak fields, a valid approach, as we will see below, for a single quark at the end of a string.

We will derive the average-field equations $\mathbf{A}^a_\mu(x)$ by varying S_{eff} while holding the ρ scale constant. This means that the vector potentials $\delta\,\mathbf{A}^a_\mu(x)$ must be varied in such a way that their variation would be orthogonal to an infinitesimal variation of the string radius $\delta\rho \mathbf{A}_\mu^a$:

$$\delta_\rho A_\mu^a (r, z) = \delta\rho \frac{\partial}{\partial\rho} A_\mu^a\left(\frac{r}{\rho}, z\right)\Big|_{\rho=1} = -\delta\rho\, r \frac{\partial}{\partial r} A_\mu^a (r, z). \tag{6}$$

Here r is the distance from the string axis, and the z axis coincides with the string axis. We can write the orthogonality condition as

$$\int_0^\infty r dr \int dz \left[r \frac{\partial}{\partial r} A_\mu^a (r, z) \right] \delta A_\mu^a (r, z) = 0. \tag{7}$$

To take this restriction into account, we must add to $\delta S_{\rm eff}$ condition (7) with an arbitrary factor. We can therefore write the equations of motion as

$$\frac{\delta S_{eff}}{\delta A_\mu^a(x)} - \frac{2\lambda}{4\pi\alpha_s(\lambda)} g^{\mu\nu} r \frac{\partial}{\partial r} A_\nu^a (r, z) = 0. \tag{8}$$

We introduce the notation $\lambda = \rho^{-2}$. We see from the solution that ρ is used as the characteristic scale or the "string radius." According to (3), α_s is a function of ρ (for convenience, we have introduced the factor $1/2\pi\alpha_s$ in the second term).

We see that the system of equations (8) is satisfied if only two components of the vector potential are retained:

$$A_0^{(1)} (r, z) = \frac{1}{\rho} f(u); \qquad A_z^{(2)} (r, z) = \frac{1}{\rho} g(u); \qquad u = \frac{r}{\rho}. \tag{9}$$

The remaining components vanish. For this choice of A_μ^a, the nonvanishing components of the gluoelectric and gluomagnetic fields are

$$E_z^{(3)} = A_0^{(1)} A_z^{(2)} = \frac{1}{\rho^2} fg; \qquad E_r^{(1)} = -\frac{\partial A_0^{(1)}}{\partial r} = -\frac{1}{\rho^2} f'(u);$$

$$H_\varphi^{(2)} = -\frac{\partial A_z^{(2)}}{\partial r} = -\frac{1}{\rho^2} g'(u).$$

The fields $E_z^{(3)}$, $E_r^{(1)}$, and $H_\varphi^{(2)}$ must decrease with increasing r; i.e., $f'(u)$, $g'(u)$, and $f \cdot g$ tend toward zero as $u \to \infty$. We see that there is a nontrivial solution of Eqs. (8) only if $f(u)$ tends toward a finite limit as $u \to \infty$.

We write $f = \omega + \varphi(u)$, where $\varphi(u) \to 0$ as $u \to \infty$. In this notation we can reduce Eqs. (8) to

$$-\frac{1}{u} \frac{d}{du}\left(u \frac{d\varphi}{du}\right) + g^2 (\omega + \varphi) = 2u \frac{d\varphi}{du}, \tag{10}$$

$$\frac{1}{u} \frac{d}{du}\left(u \frac{dg}{du}\right) + (\omega + \varphi)^2 g = -2u \frac{dg}{du}. \tag{11}$$

Let us find an approximate analytic solution of Eqs. (10) and (11) by using the small values of α_s. We will see that $\omega \sim 1$, $g \sim \alpha_s$, and $\varphi \sim \alpha_s^2$. If we disregard $\varphi(u)$, we will find from Eq. (11) that

$$\omega^2 = 4; \quad g(u) = g_0 \exp(-u^2), \tag{12}$$

and we find from Eq. (10) that

$$\varphi(u) = \frac{\omega g_0^2}{2} \int_u^\infty \frac{dt}{t} e^{-t^2} (1 - e^{-t^2}). \tag{13}$$

The constant g_0 can be determined from the requirement that the flux density of the gluoelectric field be constant:

$$\Phi^{(3)} = \oint ds_n (E_n^{(3)} / \alpha_s).$$

Near a charge there is only a radial field $E_R^{(3)}$ which is directed along the third color axis:

$$E_R^{(3)} = \frac{\alpha_s(R)}{R^2} \frac{\nu}{2},$$

where ν is the number of quarks. The flux $\Phi^{(3)} = 2\pi\nu$, and $R^2 = r^2 + z^2$. If the flux is assumed to be conserved in an approximate way, we must use the following condition in order to determine g_0:

$$\nu = \frac{1}{\alpha_s(\rho)} \int_0^\infty r dr E_z^{(3)}(r) = \frac{1}{\alpha_s(\rho)} \int_0^\infty u du f(u) g(u). \tag{14}$$

For a single quark at the end of a string we have $\nu \sim 1$.

We thus can write the following expression to within second-order terms in α_s:

$$\alpha_s \nu = \omega \int_0^\infty u du g(u) = \frac{\omega g_0}{2}; \qquad g_0 = \alpha_s \nu. \tag{15}$$

Let us determine the tension coefficient of a string. In our normalization, the energy per unit length is given by

$$\sigma(\rho) = \frac{1}{8\pi \alpha_s(\rho)} \int d^2 r [(E_z^{(3)})^2 + (E_r^{(1)})^2 + (H_\varphi^{(2)})^2]. \tag{16}$$

Using the solution for small values of $\alpha_s(\rho)$, we find

$$\sigma(\rho) = \frac{3\nu^2 \alpha_s(\rho)}{8\rho^2}. \tag{17}$$

To determine the string tension, we must minimize the energy $\sigma(\rho)$ with respect to ρ, which leads us to the equation

$$\left. \frac{\partial \ln \alpha_s(\rho)}{\partial \ln \rho} \right|_{\rho = \rho_0} = 2. \tag{18}$$

From this equation we can determine the string radius.

To determine the ρ scale, we must find $\alpha_s(\rho)$ from expression (3) for small ρ for the case in which $\epsilon(x)$ is determined by perturbation theory. In **k** representation we have $\epsilon(k) = (b/4\pi)\ln(k^2/\Lambda^2)$. Substituting this expression into (3), we find

$$\frac{4}{\alpha_s(\rho)} = \frac{b}{4\pi}\ln\left(\frac{c_1}{\Lambda^2\rho^2}\right) \equiv \frac{b}{4\pi}\ln\left(\frac{1}{\Lambda^2\rho_1^2}\right);$$

here $c_1 = 2\exp(1/3 - C) \simeq 1.57$ (C is Euler's constant). This coefficient determines the ρ scale. Since $\alpha_s(\rho_1)$ presumably increases sharply (much faster than ρ^2) when $\rho_1 \simeq R_c$, the maximum value of ρ_1 must be equal to R_c and $\rho_0^2 \simeq c_1 R_c^2$. Substituting into (17) the value of ρ obtained by us, we estimate the tension coefficient of the string to be

$$\sigma R_c^2 \simeq \frac{3\nu^2\alpha_s(R_c)}{8c_1}$$

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