Pion-baryon interactions in a soliton chiral bag model

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It is shown that *p*-wave pions interact with a nucleon at the surface of the quark core which in this model may be much smaller than the proton charge radius.

The mean square charge radius of a proton, r_p, and the mean square baryon radius of a nucleon, r_B, are on the order of 0.7 fm. In a bag model, this value corresponds to the radius of a nucleon bag on the order of 1 fm. In a chiral bag model,[1] the bag surface is a pion source and a bag radius of this magnitude complicates, for example, the description of the properties of a two-nucleon system, which suggest a radius probably on the order of 0.5 fm.[2]

The contradiction that arises can be resolved by a soliton chiral bag model,[3] in which the quark core ($R \sim 0.5$ fm) is surrounded by a topologically nontrivial configuration of a classical pion field—a chiral soliton which carries a part of the charge and the baryon distributions of a nucleon, so that $r_p \sim r_B \sim 0.7$ fm—and in which the pion source is, as usual, the bag surface. These are the main assertions upon which this study is based. The starting point is the Lagrangian of the chiral bag model (we are ignoring the gluons), which we can write as

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$$\mathcal{L} = \theta_R(x)\left[\frac{i}{2}\ \bar{\psi}\gamma\cdot\overleftrightarrow{\partial}\psi - B\right] + \frac{1}{2}\ \bar{\psi}\, U\psi\,\delta_s(x) + (1-\theta_R(x))\mathrm{Sp}\ \{+\frac{f^2}{16}L_\mu L_\mu + \frac{1}{128e^2}\,[L_\mu, L_\nu]^2\}\ , \tag{1}$$

$$L_\mu = U^+\partial_\mu U, \qquad U = \exp(i\gamma_5\vec{\tau}\vec{\Phi}), \qquad \frac{1}{2}\vec{\Phi} = \vec{\varphi}/2f.$$

Here $\theta_R(x) = \{^{1\ \text{inside the bag,}}_{0\ \text{outside the bag,}}$ $\partial_\mu\theta_R = -n_\mu\delta_s(x)$, $f = f_{\pi\to\mu\nu} = 93$ MeV, ψ is the quark field, φ is the pion field, and B is the external pressure.

In addition to the kinetic term, the chiral Lagrangian contains Skyrme's term[4] which stabilizes the chiral soliton if the bag radius is small ($2fR \ll 1$).

The static soliton solution of U_0 is [4]

$$U_0(\mathbf{r}) = \exp(i\gamma_5\vec{\tau}\mathbf{n}\,\theta(r)) \quad (\mathbf{n} = \mathbf{r}/r), \tag{2}$$

and the wave functions of quarks, ψ_0, in such an external field, U_0, are the eigenstates of the operators K^2 and K_3, where $\mathbf{K} = \mathbf{J} + \mathbf{T}$ (J is the total angular momentum, and T is the isospin).[5]

A remarkable feature of this model is that it becomes the Skyrme-Witten model for baryons in the limit $R \to 0$, with $\theta \to \pi$. The controlling factor in this case is the polarization of the quark vacuum by the soliton field U_0.[5]

These effects are closely related to the transfer of an axial charge from the bag to the external region as $R \to 0$. This charge is attributable to the continuous axial flow along the bag surface:

$$n_\mu J_\mu^{A,i}(\psi) = n_\mu J_\mu^{A,i}(U)\ . \tag{3}$$

To construct baryons with a particular spin and isospin in the Skyrme-Witten model, we must give the static soliton some rotation:

$$U(\mathbf{r},t) = A^+(t)\,U_0(\mathbf{r})\,A(t) = \exp(i\gamma_5\vec{\tau}\mathbf{t}\,\theta)\ , \tag{4}$$

where $A = a_0 + i\mathbf{a}\vec{\tau}$, and $a_0^2 + \mathbf{a}^2 = 1$, and then quantize the collective motion.[4]

It is easy to see that $t_i = R_{ij}n_j$, where

$$R_{ij} = (1 - 2\mathbf{a}^2)\delta_{ij} + 2a_ia_j + 2a_0\epsilon_{ijk}a_k\ . \tag{5}$$

If $R \neq 0$, we can transform the quark field:

$$\psi = A\,\psi'. \tag{6}$$

The boundary conditions for the fields ψ_0 and ψ' are identical in this case, but an interaction between the ψ' quarks and the collective degrees of freedom A arises in the Lagrangian

$$\mathcal{L}_{\text{int}}(\psi', A) = \psi'^+\tau_i\psi'\frac{1}{2\lambda}S_i, \tag{7}$$

where $S_i = (1/2)(a_i\pi_0 - a_0\pi_i - \epsilon_{ijk}\pi_k)$, S_i is the spin operator, and $\pi = (1/4\lambda)a$. The parameter λ appears in the soliton-rotation Hamiltonian

$$H_r = \frac{1}{8\lambda}\sum_\mu \pi_\mu^2 \,. \tag{8}$$

Since interaction (7) contributes to the baryon mass only in second-order perturbation theory, we find that $3/2\lambda = m_\Delta - m_N$. If the corrections of order $(m_\Delta - m_N)/m_N$ are ignored in the wave function of ψ', we will have $\psi' \simeq \psi_0$ and the wave functions of baryons will be the eigenstates of Hamiltonian (8).

Let us now consider the interaction of pions with baryons. The pions $\vec{\varphi}$ are the soliton fluctuations $U(U \to U^L)$:

$$U^L = LU, \quad L = 1 + i\gamma_5 \vec{\tau}(\vec{\varphi} - \langle\vec{\varphi}\rangle) f^{-1} + \dots \,. \tag{9}$$

We have taken into account here that as $r \to \infty$,

$$U = 1 + i\gamma_5 \vec{\tau} \langle \vec{\varphi} \rangle f^{-1} + \dots, \tag{10}$$

where $\langle\vec{\varphi}\rangle = 3g_A(8\pi f)^{-1}\mathbf{t} r^{-2}$ (g_A is the axial nucleon charge).

Incorporating the equations of motion and the boundary conditions for U and ψ, we can write [to within terms $\sim(m_\Delta - m_N)m_N^{-1}$] the interaction Lagrangian $\mathscr{L}_{\text{int}}(\varphi)$, linear in $\vec{\varphi}$, as

$$\mathcal{L}_{\text{int}}(\varphi) = \delta(r-R)\frac{d\langle\vec{\varphi}\rangle}{dr}\vec{\varphi}. \tag{11}$$

We ultimately find

$$\mathcal{L}_{\text{int}}(\varphi) = -\delta(r-R)3g_A(4\pi f R^3)^{-1} R_{ij}\varphi_i n_j, \tag{12}$$

where, for example,[4]

$$\langle N' | R_{ij} | N \rangle = -\frac{1}{3}\langle N' | \sigma_j \tau_i | N \rangle . \tag{13}$$

After substituting the wave functions of pions, with allowance in (12) for the fact that the bag is opaque to them,[1] and then integrating, we find

$$\int \mathcal{L}_{\text{int}}(\varphi)\, d^4x = 2\pi\delta(E_i - E_f) 3g_A (2f)^{-1} R_{ij} k_j$$
$$\times [\theta_1(kR) P_{k,0,i}(2\pi)^{-3/2}(2\omega_k)^{-1/2} + \text{H.c.}] , \tag{14}$$

where $\theta_1(x) = e^{-ix}(1 - ix - 0.5x^2)^{-1}$, and $P_{k,0,i}$ is the annihilation operator of a p-wave pion (k is the momentum modulus, and i is the isotopic index). The quadratic interaction in φ, which occurs in the external soliton region, is

$$\mathcal{L}_{\text{int}}(\varphi^2) = (1 - \theta_R(x))\frac{1}{4}\left[\sin^2\theta + \frac{1}{e^2 f^2}\left(\theta'^2 + \frac{\sin^2\theta}{r^2}\right)\right]$$
$$\times [\mathbf{t} \times \partial_\mu \mathbf{t}] \times [\vec{\varphi} \times \partial_\mu \vec{\varphi}] = (1 - \theta_R(x))\frac{1}{2f^2} J_\mu^{\upsilon,i}(U) J_\mu^{\upsilon,i}(\varphi). \tag{15}$$

Equations (14) and (15) are the main results of this study.

An unambiguous answer concerning the radius of the nucleon quark bag cannot be given because of the absence of reliable data on the πNN form factor.[2]

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