Fields as excitations of quantized coordinates

E. S. Fradkin and A. A. Tseĭtlin
P. N. Lebedev Physics Institute, Academy of Sciences of the USSR, Moscow

(Submitted 8 January 1985)
Pis'ma Zh. Eksp. Teor. Fiz. **41**, No. 4, 169–171 (25 February 1985)

A method is proposed for constructing an effective action for fields corresponding to excitations of the quantized coordinates of extended objects (strings, membranes, etc.). In the low-energy limit, the effective action includes kinetic terms for the scalar field, the gravitational field, and the field of an antisymmetric tensor. The ground state of the theory is determined by the nontrivial vacuum values of these fields (so that, in particular, the tachyon problem in the theory of a Bose string is solved).

Attempts to construct a unified theory of the interaction of elementary particles (which incorporates a quantized gravitation in a noncontradictory manner) based on the theory of strings[1] and superstrings[2] have recently attracted much interest. In the present letter we propose a new approach to the formulation of such a theory. This new approach is based not on the amplitudes on the mass shell but on a covariant effective action of the entire (infinite) set of fields.

The basic idea is that the coordinates of a D-dimensional space-time, M^D, constitute a quantized "protofield" in the theory. All the fundamental fields correspond to elementary "excitations" of this protofield, being local functions of the quantized coordinates (and having a tensor structure corresponding to the spin of the excitations).[1)] As "coordinate operator" we can take variable strings (or membranes) $x^i(z)$, $i = 1,..., D$, which correspond to a known covariant action[4,5] [an integral over a two-dimensional (or three-dimensional) internal space $\{z^\mu\}$]. The classical space-time coordinates y^i appear as "average" quantized coordinates [as a part of $x^i(z)$ which does not depend on the internal coordinates z^μ]. A covariant effective action Γ for the fundamental fields is defined by a path integral over $x^i(z)$ (and also over the metrics on the internal space). The action I, which appears in an exponent, includes the free action of the string, I_0, and also all possible covariant terms with "sources" $\left[\Sigma_n \int d^2z \partial x^{i_1}...\partial x^{i_n} B_{i_1...i_n}(x(z))\right]$. The fields $B_{i_1...i_n}$ [which are local functions of $x^i(z)$] are functional arguments of Γ. The vacuum values of the fields are found from the condition for the vanishing of the first variation of Γ with respect to all fields. The stability of the vacuum is associated with the second derivative of Γ. The higher-order derivatives of Γ at the vacuum point determine the scattering amplitudes against the background of the stable vacuum. The entire program is extended to the "supercase" by replacing the coordinates (x^i) by the supercoordinates (x^i, θ^α) (by taking the transition to a superstring).

Let us examine the calculation of Γ for the case of closed boson strings with a free action[4] $I_0 = (1/4\pi\alpha')\int d^2z\sqrt{g}g^{\mu\nu}\partial_\mu x^i\partial_\nu x^i$ ($g_{\mu\nu}$ is the metric on the two-dimensional internal space M^2; $\mu, \nu = 1, 2$). The covariant effective action Γ is defined by

$$\Gamma[\Phi, G_{ij}, A_{ij}, \dots] = \sum_{\chi} e^{\sigma\chi} \int_{M^2_\chi} [dg_{\mu\nu}] \int [dx^i] \exp\left(-\frac{I}{\hbar}\right), \tag{1}$$

$$I = \int d^2z \sqrt{g}\,\Phi(x(z)) + \frac{1}{4\pi\alpha'} \int d^2z \{\sqrt{g}\, g^{\mu\nu}\, \partial_\mu x^i \partial_\nu x^j G_{ij}(x(z))$$

$$+ \epsilon^{\mu\nu} \partial_\mu x^i \partial_\nu x^j A_{ij}(x(z))\} + \dots , \tag{2}$$

where Φ, G_{ij}, and A_{ij} are respectively the scalar field, the gravitational field (the metric), and the antisymmetric tensor field. The leading terms in (2) correspond to fields with "high spins" ($s > 2$) $(\partial x^{i_1} \dots \partial x^{i_n} B_{i_1 \dots i_n})$; $[dx^i] = \Pi_z\, dx^i \sqrt{G}$, $G = \det G_{ij}$. The summation runs over the Euler characteristic of the closed compact space M^2, i.e., over the number of "handles," n ($\chi = 2 - 2n$); $\sigma =$ const. The invariance of (2) under general covariant transformations in M^D and under gauge transformations $\partial A_{ij} = \partial_i \lambda_j - \partial_j \lambda_i$ renders G_{ij} and A_{ij} "massless" in Γ; all other fields are massive. The action I_0 is invariant under the substitution $x^i \to x^i +$ const, so that the "partition function" of the free string $\Gamma[0, G_{ij} = \delta_{ij}, 0, \dots]$ contains a volume factor $\int d^D y$. In the presence of fields, this integral over the "null mode" is no longer trivial; i.e., Γ is given by an integral over the space-time M^D. An integral over the classical "coordinates" y^i is selected in the standard way: $x^i(z) = y^i + u^i(z)$, $\int dx F[x] = \int d^D y \int [du]F[y+u]$, $[du] = du\,\delta^{(D)}(P^i(y,u))Q(y,u)$. The value $P = 0$ violates the $u \to u +$ const invariance, and we have $Q = \det(\partial P(u+a)/\partial a)|_{a=0}$. As a result, Γ becomes

$$\Gamma = \int d^D y \sqrt{G(y)}\, \mathcal{L}\,(\Phi(y), D_i \Phi(y), \dots ;\; G_{ij}(y), R^i_{jkl}(y), \dots ;\; F_{ijk}(y), \dots), \tag{3}$$

where D_i is a covariant derivative with respect to G_{ij}, $R^i_{\ jkl}$ is the curvature tensor of G_{ij}, and F_{ijk} is the field strength of A_{ij}. An expansion in $\alpha' \to 0$ corresponds to an expansion in the loops of the integral over u^i. To obtain a covariant perturbation theory we can use geodesic coordinates near the point y^i, replacing u^i by $\sigma^i = u^i - \frac{1}{2}\Gamma^i_{jk}(y)\, u^j u^k + \dots$, and choosing $P^i = \int d^2 z \sqrt{g}\sigma^i$. Retaining only the first term (with $\chi = 2$) in sum (1) (the "tree approximation" in string theory), and evaluating the integral over the metrics in the semiclassical approximation (i.e., integrating over the closed surfaces of M^2, which "differ only slightly" from a sphere), we find

$$\Gamma \simeq -c\,(2\pi\alpha')^{-D/2} \int d^D y \sqrt{G}\,\{\Omega^2 - \alpha'(\partial_i \Omega)^2 (a_1 + a_2 \ln \Omega)$$

$$+ \alpha'\left(R - \frac{1}{4} F_{ijk} F^{ijk}\right)(b_1 + b_2 \ln \Omega) + O(\alpha'^2)\} , \tag{4}$$

$$a_1 = \nu(5 + 2\nu)(1+\nu)^{-2},\; a_2 = -2\nu(1+\nu)^{-3},\; b_1 = \frac{1}{4},\; b_2 = -\frac{1}{2}(1+\nu)^{-1},$$

where $\nu = (1/6)(D - 25)$ if $D < 26$ or $\nu = 1$ if $D = 26$, and $\Omega = (\Lambda^{-2}\nu^{-1}\Phi)^\gamma$, where $\gamma = -(1+\nu)/2$ ($\Lambda \to \infty$ is the cutoff in the two-dimensional theory). The ground state of the theory is determined by the classical equations corresponding to (4). Assuming a maximum symmetry of the vacuum ($\Omega =$ const, $B_{ijkl\dots} = 0$), we find

$$R - \frac{1}{4} F_{ijk} F^{ijk} = \mu^2 N_1 ,\quad D_i F^{ijk} = 0,$$

$$R_{ij} - \frac{3}{4}F_{imn}F_j{}^{mn} = \mu^2 N_2 G_{ij}, \ \mu^2 = 1/\alpha', \tag{5}$$

$$N_1 = -2(b_2 + 2\Delta)^{-1}, \ N_2 = b_2[2\Delta(b_2 + 2\Delta)]^{-1}, \quad \Delta \equiv b_1 + b_2 \ln \Omega \ .$$

The general solution of (5) corresponds to the distinction of three-dimensional subspaces by virtue of $F_{ijk} \sim \epsilon_{ijk}$ (cf. Ref. 6; there are no solutions with a plane space M^D). In this case we have

$$R_{ab} = \mu^2 N_3 G_{ab}, \quad R_{\alpha\beta} = \mu^2 N_2 G_{\alpha\beta}, \quad F_{ijk}F^{ijk} = 4(N_3 - N_2),$$
$$N_3 = [4\Delta - (D-2)b_2][4\Delta(b_2 + 2\Delta)]^{-1}; \qquad a, b = 1,2,3, \quad \alpha, \beta = 4, ..., D. \tag{6}$$

Solutions (6) correspond to an "effective" space-time in the form of a three-dimensional (anti-) de Sitter "multiplied" by a $(D-3)$-dimensional internal space (e.g., S^{D-3}). Equations (5) also allow solutions with an "internal" space in the form of a product of 3-spheres, $S^3 \times ... \times S^3$ (in this case we have $F_{ijk} \sim \epsilon_{ijk}$ for each of the spheres). Some of solutions (6) are stable, telling us that the tachyon problem in the boson-string theory can be solved through the formation of nontrivial background values of the scalar, gravitational, and antisymmetric tensor fields corresponding to the true vacuum of the theory.

Finding the vacuum values of the fields, we may ask whether the amplitudes calculated against the background of this vacuum have reasonable physical properties; i.e., is the theory of a string against the nontrivial background noncontradictory? One necessary condition for consistency (the absence of "ghosts") is the conformal invariance of the two-dimensional theory with action (2) taken for the vacuum values Φ, G_{ij}, A_{ij},... . For the vacuum of the boson-string theory found above ($G_{ij} \sim$ metric of $S^3 \times ...$, $F_{ijk} \sim \epsilon_{ijk}$), action (2) incorporates (for each S^3 factor) an action for the SU(2)-sigma model with a Wess-Zumino term with a relation among constants corresponding to a zero of the β function, i.e., a conformally invariant theory.[7] The requirement of conformal invariance may make it possible to find the ground state without expanding Γ in $\alpha' \to 0$. An analysis of compactification on the basis of an expansion of Γ in α' is valid only if the size (r) of the compact dimensions is substantially larger than $\sqrt{\alpha'}$ (the size of the string); i.e., we are dealing with a problem of finding a natural hierarchy $r \gg \sqrt{\alpha'}$. A method that might be used to calculate $\Gamma(1)$ without an expansion in α' would be to specify a certain *ansatz* for the vacuum values of the fields and to expand Γ in powers of the deviations of the fields from their vacuum values (the semiclassical calculation of the corresponding coefficients, which give the amplitudes against the background of the nontrivial vacuum, would incorporate the coefficients of the exponential functions).

In summary, this new approach makes it possible to correctly formulate and to solve the problem of finding the ground state in a string theory. It would thus be of particular interest to generalize this approach to superstring theory,[2] in which the ground state should correspond to six compact spatial dimensions.

[1)]The concept of a noncommutativity of coordinates with fields was first introduced by Markov.[3]

[1]J. Scherk and J. H. Schwarz, Nucl. Phys. **B81**, 118 (1974).
[2]J. H. Schwarz, Phys. Rep. **89**, 223 (1982); M. B. Green and J. H. Schwarz, Preprint CALT-68-1194, 1984; Phys. Lett. **149B**, 117 (1984).
[3]M. A. Markov, Zh. Eksp. Teor. Fiz. **10**, 1311 (1940).
[4]L. Brink, P. Di Vecchia, and P. S. Howe, Phys. Lett. **65B**, 471 (1976); A. M. Polyakov, Phys. Lett. **103B**, 207 (1981).
[5]A. Sugamoto, Nucl. Phys. **B215**, 381 (1983).
[6]P. G. O. Freund and M. A. Rubin, Phys. Lett. **97B**, 223 (1980).
[7]E. Witten, Commun. Math. Phys. **92**, 455 (1984).

Translated by Dave Parsons