Spin relaxation of an electron system in which the spectrum degeneracy is lifted

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The spin-relaxation mechanism in a quantizing magnetic field H is analyzed. This mechanism is peculiar to electron systems in which the degeneracy of the spectrum has been lifted. In crystals such as wurtzite, the relaxation time, measured in picoseconds, has an enormous anisotropy in the H direction and an unusual field dependence.

In the present letter we analyze a mechanism which leads in the case of Landau quantization to a very fast spin relaxation of the electron system in which the Kramers degeneracy of the spectrum has been lifted. We will show that the relaxation linked with this mechanism, which has previously not been discussed in the literature, has several unusual properties.

A simple example of this system are the electrons in crystals such as wurtzite with a symmetry group C_{6v} (to which CdS and CdSe, for example, belong). In crystals of this sort, the behavior of electrons near the conduction-band edge is determined by the Hamiltonian[1]

$$H_e = H_0 + H_{so} = \frac{\hbar^2 k^2}{2m} + \frac{1}{2} g\mu_B \vec{\sigma}\,\mathbf{H} + \lambda\vec{\nu}\,[\vec{\sigma}\times\mathbf{k}\,], \tag{1}$$

where $\hbar\mathbf{k}$ is the operator of the kinematic momentum in a magnetic field, g is the spectroscopic-splitting factor, μ_B is the Bohr magneton, $\vec{\sigma}$ are the Pauli matrices, λ is a constant which determines the extent of the spin-orbit splitting of the conduction band at $\mathbf{k}\neq 0$ in the absence of a field, and $\vec{\nu}$ is a unit vector directed along the hexagonal C_6 axis. Since a Hamiltonian such as $H_{so} = \lambda\vec{\nu}[\vec{\sigma}\times\mathbf{k}]$ appears in several other problems, in particular, in the description of spin-orbit interaction in $2d$ electron systems,[2] the model under consideration is of a more general interest.

In the absence of a magnetic field, the interaction H_{so} is, as indicated in Ref. 3, responsible for the appearance of a spin-relaxation precession mechanism which is analogous to the familiar D'yakonov-Perel' mechanism. In a field in which the Zeeman level splitting greatly exceeds the spin-orbit splitting, this interaction plays a different role, establishing an effective coupling between the electronic spins and the lattice. This capability of crystals with a T_d symmetry group, in which $H_{so} \propto k^3$, was initially pointed out by Rashba (see footnote 3 in Ref. 4). We will use this idea to show how the spin-lattice interaction arises in the model that we are analyzing. We will introduce into original Hamiltonian (1) a perturbation H_{e-d} caused by the lattice defects. We will later define H_{e-d} concretely, after examining the more important interactions that occur at low temperatures—the interaction with ionized impurities (e-i) and

the interaction with piezoacoustic phonons (e-p). The term H_{so} in the complete Hamiltonian $H = H_0 + H_{so} + H_{e-d}$ can be eliminated in first order in λ by means of a canonical transformation $\tilde{H} = e^{-S} H e^S$, where S is given in symbolic operator notation by

$$S = -i \int_0^\infty d\tau\, e^{-\delta\tau} \exp\left(iH_0\tau\right) H_{so} \exp(-iH_0\tau), \quad \delta \to 0. \tag{2}$$

As a result, we can single out from $\tilde{H}$ the operator $H_{s-d} = [S, H_{e-d}]$ which generates spin-flip transitions due to electron scattering.

Let us consider the transitions between the upper $(s = \uparrow)$ and the lower $(s = \downarrow)$ spin subbands of zero Landau band under the assumption that the electrons which are photoexcited to the conduction band after being thermalized in a time $\tau_e < T_1$ are near the bottom of the upper subband. The spin-lattice-relaxation time T_1 is determined in this case by

$$\frac{1}{T_1} = \int_0^\infty dk_\parallel\, W(k_\parallel)\, f(k_\parallel) \;/\; \int_0^\infty dk_\parallel\, f(k_\parallel), \tag{3}$$

where $\hbar k_\parallel$ is the projection of the electron momentum in the direction of $\mathbf{H}$, $W(k_\parallel)$ is the probability for the spin-flip transition, $f(k_\parallel) = \exp(-k_\parallel^2/k_T^2)$ is the Boltzmann factor, and $\hbar k_T$ is the thermal momentum.

To calculate T_1^{-1} for the general case of arbitrary orientation of the magnetic field $\mathbf{H}$ with respect to the x_0, y_0, z_0 coordinate system with the z_0 axis parallel to C_6, we must transform to an x, y, z, coordinate system (with the z axis parallel to $\mathbf{H}$), in which the eigenstates of the Hamiltonian H_0 form a Landau basis $|n, k_x, k_z, s\rangle$. In this transformation, the same as that of Rashba and Sheka,[5] we switch to new components of the momentum and spin operators by means of the $R \epsilon SO(3)$ and $D \epsilon SU(2)$ matrices, respectively. Each matrix is parametrized by Euler's angles $\Phi = \varphi + \pi/2$ and $\Theta = \theta$, $\Psi = 0$, where θ and φ are angles that determine the direction of $\mathbf{H}$ in the x_0, y_0, z_0 coordinate system. Using (1) and (2), we thus find the following expression for the matrix element of the spin-impurity interaction, H_{s-i}:

$$M_q = i\frac{\lambda k_H}{\hbar\omega_c} C_q \left(\frac{\sqrt{2}}{2-\eta}\, e^{i\psi} \kappa_\perp \cos\theta - \frac{i}{\eta} \kappa_\parallel \sin\theta\right) \exp\left(-\frac{\kappa_\perp^2}{2}\right) \delta_{k_x',\, k_x+q_x}\, \delta_{k_z',\, k_z+q_z}. \tag{4}$$

Here

$$\kappa_\perp = \frac{q_\perp}{\sqrt{2}k_H}, \quad \kappa_\parallel = \frac{q_\parallel}{k_H}, \quad \eta = \frac{\omega_s}{\omega_c} = \frac{m}{m_s}, \quad m_s = \frac{2m_0}{g}, \tag{5}$$

ω_c and ω_s are the cyclotron and Zeeman frequencies, m_s is the "spin" mass, m_0 is the free-electron mass, $\hbar k_H$ is the magnetic momentum, C_q is the Fourier transform of the screened Coulomb potential, and $q_\perp$ and ψ are the polar coordinates in the q_x, q_y plane. Using expressions (3)–(5), we find

$$\frac{1}{T_{1i}} = 8\pi \frac{N_i \, m \, e^4}{\epsilon_0^2 \, \hbar^3 \, k_H^3} \left(\frac{m_s}{2m}\right)^{1/2} \left(\frac{\lambda k_H}{\hbar \omega_c}\right)^2 F_i(\theta), \tag{6}$$

where

$$F_i(\theta) = \frac{K_1(\zeta)}{\eta} \sin^2\theta + \frac{K_2(\zeta)}{(1-\eta)^2} \cos^2\theta, \qquad \zeta = \eta + \frac{q_s^2}{k_H^2} \tag{7}$$

$$K_1(\zeta) = e^\zeta Ei(-\zeta) + \zeta^{-1}, \qquad K_2(\zeta) = - \left[1 + (1+\zeta) e^\zeta Ei(-\zeta) \right]; \tag{8}$$

N_i is the impurity concentration, ϵ_0 is the dielectric constant of the crystal, e is the elementary charge, $Ei(-\zeta)$ is the exponential integral, and q_s is the reciprocal screening radius. Wallace[6] showed that $q_s \propto k_H$ in the ultraquantum limit, and hence ζ does not depend on H.

Let us now consider the case of piezoacoustic scattering. We disregard the crystal anisotropy, since it is much weaker than the anisotropy caused by spin-orbit interaction. Assuming that electron scattering is elastic, a justifiable assumption under the condition $k_H \ll T/\hbar v$ (T is the temperature, and v is the velocity of sound), and using the same procedure as in the preceding case, we find

$$\frac{1}{T_{1p}} = 16\pi \frac{m \, e^2 \beta^2}{\rho v \epsilon_0^2 \, \hbar^2} \left(\frac{m_s}{2m}\right)^{1/2} \left(\frac{T}{\hbar v k_H}\right) \left(\frac{\lambda k_H}{\hbar \omega_c}\right)^2 F_p(\theta), \tag{9}$$

where ρ is the crystal density, β is the piezoelectric modulus, and the expression for $F_p(\theta)$ is derived from (7) through the substitution $\zeta \rightarrow \eta$.

Before analyzing the results, we should mention that according to Eqs. (6)–(9), $T_{1i} \propto H^{5/2}$ and $T_{1p} \propto H^{3/2}$. A field dependence of this kind is qualitatively different from the dependence $T_1 \propto H^{-\alpha} (\alpha > 0)$, which we find from the currently discussed mechanisms of spin relaxation in crystals without an inversion center.[7] The difference stems from a Hamiltonian which is substantially different from that in Ref. 7 and which is responsible for the relaxation.

The most intriguing feature in the relations obtained by us is the strong dependence of T_1 on the angle between the directions of $\mathbf{H}$ and C_6, the crystal axis. If the conditions $\eta \ll 1$ and $\zeta \ll 1$ hold, the ratio $T_1^\parallel / T_1^\perp$ for the two field orientations ($\mathbf{H} \parallel \vec{v}$ and $\mathbf{H} \perp \vec{v}$) is large in the parameters $1/\eta\zeta$ and $1/\eta^2$ in the case of e-i scattering and e-p scattering, respectively. Numerical estimates of CdS ($m = 0.2m_0$, $g = 1.8$) give $T_{1i}^\parallel / T_{1i}^\perp \approx T_{1p}^\parallel / T_{1p}^\perp \approx 20$; for CdSe ($m = 0.013m_0$, $g = 0.5$) we have $T_{1i}^\parallel / T_{1i}^\perp \approx 200$ and $T_{1p}^\parallel / T_{1p}^\perp \approx 450$. We see that the anisotropy is indeed enormous.

Let us finally determine the order of magnitude of T_1 in CdS by setting $H = 80$ kOe ($\mathbf{H} \perp \vec{v}$), $T = 4.2$ K, and $N_i = 5 \times 10^{15}$ cm^{-3}. Assuming that $v \sim 10^5$ cm/s, $\epsilon_0 \sim 10$, $\beta \sim 10^5$ esu, and $\lambda = 1.6 \times 10^{-10}$ eV·cm (Ref. 8), we find $T_{1i}^\perp \approx 2$ ps and $T_{1p}^\perp \approx 5$ ps. Note that under the same conditions the momentum-relaxation time is $\tau_p \sim 0.1$ ps (Ref. 9). The usual hierarchy of electronic relaxation times thus applies in this case: $\tau_p \ll T_1$. We point out, for comparison, that the spin-relaxation times obtained by us for CdS are three to four orders of magnitude shorter than those for InSb, a material with

the strongest spin-orbit coupling in the A_3B_5 compounds. These estimates show that at low temperatures the relaxation mechanism proposed by us plays a dominant role in establishing a spin equilibrium in the systems under consideration.

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