Electron spin susceptibility of superconductors

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(Submitted 7 January 1985)
Pis'ma Zh. Eksp. Teor. Fiz. **41**, No. 5, 188–191 (10 March 1985)

The effect of spin polarization due to the Meissner currents on the electron spin susceptibility of a superconductor is studied. This effect accounts for a susceptibility considerably stronger than that of a normal metal. The spin distribution is discussed.

According to the Bardeen-Cooper-Schrieffer theory, the spin susceptibility of the electrons of a superconductor approaches zero at low temperatures.[1] If, however, the spin-orbit interaction with impurities is taken into account, the susceptibility of a normal metal will be nonvanishing, although lower than χ_0, even at $T = 0$.[2] We wish to point out a spin-susceptibility mechanism which is present only in superconductors and which accounts for susceptibilities higher than χ_0. This mechanism, which is analogous to the current-induced spin-polarization mechanism in a normal metal,[3] involves the polarization of the spins of electrons that produce the Meissner current during spin-orbit scattering by impurities.

To calculate this contribution to the spin response, which we call the anomalous susceptibility, we must analyze diagrams of the type in Fig. 1. In this diagram a cross denotes the total scattering amplitude with allowance for spin-orbit interaction and the line with an arrow represents a Green's function in the BCS model. The diagrams should be averaged over the impurities in the usual manner and the odd part with respect to k, which gives us the magnetization component, $\mathbf{M}_\mathbf{k} = \chi_1(\mathbf{k})\mathbf{H}_\mathbf{k}$, should be singled out. To avoid obtaining a trivial result from the diagram in the summation over the frequencies and in calculating the trace of the product of the Pauli matrices, it is important to choose a scattering amplitude which contains an odd part with respect to frequency, in addition to a spin-dependent part. For model-based calculations, we chose point impurities; the amplitude for the scattering by these impurities can be calculated in second Born approximation:

$$T_{pp'}(\omega) = u + i\,\frac{u\Lambda}{p_0^2}\,(\vec{\sigma},\ \mathbf{p}\times\mathbf{p}') - i\,\frac{p_0 m}{2\pi}\,u^2\,\mathrm{sgn}\,\omega\,.$$

Here Λ is the spin-orbit-interaction constant $[\Lambda \sim (Ze^2/\hbar c)^2]$. The calculation result for

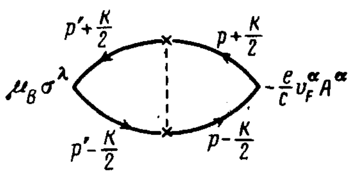

FIG. 1

a pure superconductor ($\xi_0 \ll l$) is

$$\chi_1(\mathbf{k}) = -\left(\frac{p_0^2}{2\pi\hbar^2}\right)^2 \mu_B \frac{e}{c} \frac{u\Lambda}{2\tau} T \sum_\omega \frac{\Delta^2}{(\omega^2+\Delta^2)^{5/2}} Q\left(\frac{k v_F}{2(\omega^2+\Delta^2)^{1/2}}\right), \tag{1}$$

where $Q(x) = x^{-6}(x - \arctan x)[(x^2+1)\arctan x - x]$, and the calculation result for an impure superconductor ($\xi_0 \gg l$) is

$$\chi_1(\mathbf{k}) = -\left(\frac{p_0^2}{3\pi\hbar^2}\right)^2 \mu_B \frac{e}{c} u\Lambda\tau\, T \sum_\omega \frac{\Delta^2}{\omega^2+\Delta^2} \frac{1}{(\omega^2+\Delta^2)^{1/2} + (2\tau_s)^{-1} + \frac{1}{2} Dk^2} \tag{2}$$

$\chi_1(0)$ are estimated to be

$$\chi_0 \Lambda \xi_0^2 p_0 / l,\ \xi_0 \ll l; \qquad \chi_0 \Lambda p_0 l \frac{\Delta_0}{\Delta_0 + \tau_s^{-1}}, \quad \xi_0 \gg l. \tag{3}$$

If the spin-orbit-interaction constant is appreciable—it can be on the order of unity in superconductors with a large atomic number (Pb, Sn, Hg)—the anomalous susceptibility can be much larger than the usual one, as is evident from (3). Furthermore, we see from (1) and (2) that its arbitrary sign is determined by the sign of Λ and by the impurity potential, which may be either attractive or repulsive.

A large value of $\chi_1(0)$, however, does not necessarily imply that an effect (a Knight shift, for example) is large. To illustrate this point, we will examine a film of thickness d in a uniform magnetic field parallel to the film surface. We assume that Δ is constant in the film. We also assume that boundary conditions for the scattering are specular and that the impurities are point impurities. Since the electrons producing the Meissner current are scattered by the impurities, the impurities become the source of the spatial separation of electrons with opposite spin polarizations. Since the current is nonuniform, the spins of electrons scattered into a single point do not cancel each other out, giving rise to a magnetization proportional to the current nonuniformity, i.e., to the field. Furthermore, the electrons scattered toward the wall are reflected by it.This means that the wall becomes a source of the spin distributed near it in a layer on the order of the correlation radius of the kernel, $\chi_1(\mathbf{k})$ [$min(\xi_0)$, $(\xi_0 l)^{1/2}$, $\Lambda^{-1} l$)]. In the case of scattering by a point source, there is no average spin (it vanishes after averaging over the scattering angle). The magnetization which is averaged over the film thickness therefore vanishes. We can use these arguments to determine the spin distribution in a film:

$$M(x) = AT \sum_\omega \frac{\Delta^2}{\omega^2+\Delta^2}\left[\frac{L(\omega)\,d}{D}\left(\exp\left(-\frac{x}{L(\omega)}\right) + \exp\left(-\frac{d-x}{L(\omega)}\right)\right) - \frac{1}{(\omega^2+\Delta^2)^{1/2} + (2\tau_s)^{-1}}\right]H,$$

$$0 \leq x \leq d,\ A = \left(\frac{p_0^2}{3\pi\hbar^3}\right)^2 \mu_B \frac{e}{c} u\Lambda\tau,\ L(\omega) = \left(\frac{D}{2(\omega^2+\Delta^2)^{1/2} + \tau_s^{-1}}\right)^{1/2} \tag{4}$$

$$L(\omega) \sim min\left((\xi_0 l)^{1/2},\ \Lambda^{-1} l\right).$$

We assume that the superconductor is impure ($\xi_0 \gg l$) and that the field is constant ($d \ll \delta_{\text{London}}$). Let us consider two limiting cases.

a) $d \gg L$,

$$M = -AT \sum_\omega \frac{\Delta^2}{\omega^2 + \Delta^2} \frac{1}{(\omega^2 + \Delta^2)^{1/2} + (2\tau_s)^{-1}} H \qquad \text{(inside the film)}$$

$$M = AdT \sum_\omega \frac{\Delta^2}{\omega^2 + \Delta^2} \frac{L(\omega)}{D} H \qquad \text{(on the film surface)} \tag{5}$$

b) $d \ll L$,

$$M(x) = A\,\Delta \tanh\left(\frac{\Delta}{2T}\right) \frac{1}{D} \left(\left(x - \frac{1}{2} d\right)^2 - \frac{1}{12} d^2 \right) \tag{6}$$

Clearly, if we will now measure the spin susceptibility (from the Knight shift, for example), the anomalous part of this susceptibility will contribute significantly when $d \gg L$ and it will broaden the NMR line only slightly when $d \lesssim L$ (the surface and bulk components of magnetization average out and cancel each other). The fact that this effect vanishes in small samples may be the reason that anomalous susceptibility has not been observed experimentally.

Let us now consider from a more general point of view the total spin moment stemming from the anomalous susceptibility. If we assume that the orbital-current and spin-moment densities are thermodynamic variables, we can introduce the fields **A** and **H** corresponding to them, which can formally be considered to be independent. A response is linear in a weak field:

$$j_\mu(x) = c \int d^3x' \left(Q^1_{\mu\nu}(x, x') A_\nu(x') + Q^2_{\mu\nu}(x, x') H_\nu(x')\right),$$

$$M_\mu(x) = \int d^3x' \left(Q^3_{\mu\nu}(x, x') A_\nu(x') + Q^4_{\mu\nu}(x, x') H_\nu(x')\right); \tag{7}$$

here $Q^1_{\mu\nu}(x,x') = Q^1_{\nu\mu}(x',x)$, $Q^4_{\mu\nu}(x,x') = Q^4_{\nu\mu}(x',x)$, and $Q^2_{\mu\nu}(x,x') = Q^3_{\nu\mu}(x'x)$. In Eq. (7) the kernel Q^1 corresponds to the Meissner effect, the kernel Q^4 corresponds to an ordinary spin susceptibility, Q^3 corresponds to an anomalous susceptibility, and Q^2 corresponds to an effect which is thermodynamically conjugate to it. We assume that **M** is the total spin moment of a sample. From (7) we can easily derive $\delta M_\mu / \delta A_\nu(x) = \int d^3x' Q^2_{\nu\mu}(x,x')$, which is, to within a constant, the orbital-current density $j_\nu(x)$ produced in a constant field $H_\lambda = \delta_{\lambda\nu}$. Far from the surface, the kernel Q^2 depends solely on the difference between its arguments, so that the current distribution $j_\nu(x)$ is uniform: $j_\nu = a_{\nu\mu} H_\mu$. Since $a_{\nu\mu}$ is a pseudotensor, it vanishes if there is an inversion in the symmetry group of the superconductor. As for the current distribution near the surface, it is generally nonvanishing even if there is an inversion. The current-distribution range depends on the properties of the surface.

All these results suggest that if the symmetry allows a second-rank pseudotensor, then the superconductor as a whole accounts for the total moment, but if the symmetry does not allow a second-rank pseudotensor, then only a region of size L near the surface contributes to the total moment. The example given above clearly shows, however, that these contributions may be of the same order of magnitude. (In this example, a cancellation of the total moment is a property of the chosen model, rather than a typical occurrence).

We emphasize again that we are concerned here solely with the anomalous part of the susceptibility which is produced by the action of the magnetic field on the orbital current.

Another characteristic effect we should point out, which occurs near the phase-transition point, is the temperature dependence of spin density near the surface of a superconductor in a magnetic field. If a sample is much larger than $\delta_{\rm London}$, then the spin density in a layer of thickness L near the surface is about $(\delta_{\rm London}/L)\,\chi_1(0)H$; i.e., it is such that there is an order-of-magnitude cancellation of the volume part of the moment. Since the spin density at the surface changes as $(T_c - T)^{1/2}$ near the phase-transition point, and since the spin density associated with ordinary susceptibility changes slower (as $T_c - T$), we can determine the anomalous part of the susceptibility from the temperature dependence of the Knight shift in the NMR of the atoms near the surface.

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Translated by S. J. Amoretty