Emission of fast electrons due to Coulomb scattering in a magnetized plasma

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Magneto-Coulombic radiation resulting from excitation of the Landau transverse levels of ultrarelativistic electrons and from the subsequent radiative transition to the ground state is studied. The magneto-Coulombic radiation in magnetic fields of dense astrophysical objects is one of the principal sources of high-energy photons.

Detection of the cyclotron line in local x-ray sources[1] and in γ-ray bursts[2] shows that the radiation originates in magnetic fields of 10^{11}–10^{13} G. Rapid rotation of dense objects with fields of this strength leads to the acceleration of electrons and positrons to ultrarelativistic energies in the direction of the magnetic field by the induced electric fields (see Ref. 3, for example). A radiation produced as a result of the interaction of fast particles with the magnetic field (synchrotron radiation[4] and flexural radiation[5]) and with matter (bremsstrahlung) has been studied in models of pulsars. In the present letter we show that a combined action of the magnetic field and matter produces radiation during the initial motion of particles along the lines of force.

In the absence of matter, an electron in a strong, uniform magnetic field rapidly goes to the Landau ground level, and then stops emitting photons. In the presence of matter, the moving particles go to the Landau excited levels (the analog of multipole scattering in a plasma without a magnetic field) and then return to lower-lying levels while emitting photons. In the electron rest frame the photon energy is $\hbar\omega_B = \hbar eB/mc$. In the laboratory frame of reference, the emitted energy is on the order of $\hbar\omega_B\gamma$, where $\gamma = E_e/mc^2$. In the fields $B \lesssim B_0 = m^2c^3/\hbar e = 4.4\times 10^{13}$ G, the energy of an emitted photon, which is comparable to the initial energy of a particle, extends into the region of hard γ radiation in the case of ultrarelativistic electrons.

The electron and ion collisions in quantizing magnetic fields were studied in Refs. 6–10. If we assume that in the electron rest frame the electron is nonrelativistic before and after the collision (which gives rise to the constraint $B \lesssim 0.1\, B_0$) and that in the laboratory frame of reference the electron moves at a relativistic velocity ($\gamma \gg 1$) in the direction of the magnetic field, we can derive for the cross section for a transition from the n level to the n' level much simpler equations than those obtained in Refs. 9 and 10, and we can also take the screening into account:

$$\sigma_{nn'} = 2\pi Z^2 r_e^2 \frac{B_0}{B} W_{nn'}(x_{nn'}). \tag{1}$$

Here r_e is the classical electron radius, Z is the nuclear charge,

$$W_{nn'}(\xi) = \frac{n!}{n'!} \int_0^\infty dx \frac{x^{n'-n} e^{-x}}{(x+\xi)^2} \left(L_n^{n'-n}(x)\right)^2 \tag{2}$$

is a function which was introduced in Ref. 8, L_n^m is a Laguerre polynomial,

$$x_{nn'} = \frac{B}{2\gamma^2 B_0}(n-n')^2 + \frac{\hbar c}{2eB\rho_D}, \tag{3}$$

and ρ_D is the Debye screening length.

We assume that the gyroradius $(\hbar c/eB)^{1/2}$ of a particle is small in comparison with ρ_D. This condition establishes the lower limit on the magnetic-field strength which we can write for the electron-proton plasma as

$$\frac{B}{B_0} \gg 4\pi n_p r_e \left(\frac{\hbar}{mc}\right)^2 \frac{mc^2}{kT} = 5.3\times 10^{-33} n_p \frac{mc^2}{kT}, \tag{4}$$

where n_p is the proton density, and T is the plasma temperature. In this case, $x_{nn'} \ll 1$. In the case of the transition $0\to 1$, we derive the following expression from Eq. (1), with allowance for the small value of x_{01}:

$$\sigma_{01} \doteq 4\pi Z^2 r_e^2 \frac{B_0}{B}\Lambda, \quad \Lambda = \frac{1}{2}\ln\frac{0.207}{x_{01}}. \tag{5}$$

The cross sections for transitions to the remaining excited levels do not contain a large logarithm. The total cross section for a transition to levels with $n' \geqslant 2$ is

$$\sum_{n'\geqslant 2} \sigma_{0n'} = \sum_{n'\geqslant 2} 2\pi Z^2 r_e^2 \frac{B_0}{B}\frac{1}{n'(n'-1)} = 2\pi Z^2 r_e^2 \frac{B_0}{B} \tag{6}$$

It is therefore sufficient to take only the transition $0\to 1$ into account.

We assume that the ionic density n_i of the plasma is such that an electron from the level $n = 1$ has time to go to the ground state before its next collision; i.e., the condition $c\sigma_{01} n_i \ll 1/\gamma\tau$, where τ is the lifetime of the $n = 1$ state in the electron rest frame, holds.[9] We can write this inequality for ($\Lambda = 10$) as

$$Z^2 n_i \left(\frac{B_0}{B}\right)^3 \gamma \ll 3\times 10^{31}. \tag{7}$$

The collisional transition $0\to 1$ and the radiative transition $1\to 0$ that follows can be treated as a single process which we call magneto-Coulombic radiation. Integrating over the photon-emission angles, we can write the cross section for magneto-Coulombic radiation as

$$\frac{d\sigma}{d\omega} = \frac{3\sigma_{01}}{2\omega_m}\left(1 - 2\frac{\omega}{\omega_m} + 2\left(\frac{\omega}{\omega_m}\right)^2\right), \tag{8}$$

where $\omega_m = 2\gamma\omega_B$ is the maximum photon frequency.

The energy of electrons lost due to magneto-Coulombic radiation is

$$-\left(\frac{dE_e}{dx}\right)_{\rm mC} = n_i \int \hbar\omega \frac{d\sigma}{d\omega} d\omega = 4\pi Z^2 r_e^2 E_e n_i \Lambda. \tag{9}$$

The energy loss (9) and the radiation loss of electrons depend on the electron energy primarily in a linear manner. The ratio of the magneto-Coulombic loss to the radiation loss is

$$\frac{(dE_e/dx)_{\rm mC}}{(dE_e/dx)_{\rm rad}} \approx \frac{\pi}{\alpha} = 137\pi . \tag{10}$$

In (10) we make no distinction between Λ and the logarithm in the radiation loss. In the case of magneto-Coulombic radiation, the analog of the radiation length is the quantity

$$l_{\rm mC} = 1/4\pi Z^2 r_e^2 n_i \Lambda \approx \frac{\alpha}{\pi} l_{\rm rad} . \tag{11}$$

The magneto-Coulombic radiation is thus one of the principal mechanisms for generating high-energy γ radiation from dense astrophysical objects with strong magnetic fields.

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