CP violation in the Kobayashi-Maskawa model with four generations

A. A. Ansel'm, T. A. Zhukovskaya, N. G. Ural'tsev, and Dzh. L. Chkareuli
B. P. Konstantinov Institute of Nuclear Physics, Academy of Sciences of the USSR

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A useful method of parametrizing the mixing matrix is proposed. It is shown that the value of ϵ and the ratio ϵ'/ϵ of either sign can easily account for the existence of the fourth generation only if the mixing in of the fourth generation with the first two generations is at least as strong as that of the third generation. In the $B_s^0 - \overline{B}_s^0$ system the fourth generation can substantially increase the CP-odd mixing.

The Kobayashi-Maskawa model with three generations of fermions is now being critically tested, primarily to see whether it can describe the CP violation in K° decays. The unexpectedly small mixing angles θ_2 and θ_3 (Ref. 1) should yield a reasonably large ratio $\epsilon'/\epsilon \gtrsim 1$–$2\%$ (Ref. 2), which may be at variance with the experimental results of Ref. 3. It is unlikely that the absolute value of ϵ can reach the experimental value at $m_t \lesssim 40$ GeV. The experiments aimed to obtain more accurate data on θ_2 and θ_3 and ϵ'/ϵ may pose serious difficulties for the standard Kobayashi-Maskawa model. In the present letter we consider a highly simplified generalization of this model—a model with four generations of quarks.

1. We will parametrize the mixing matrix V in the following way in the case of n generations of quarks:

$$V = \prod_{i=n-1}^{1} \prod_{j=i+1}^{n} V_{ij} = \begin{cases} V_{23}\, V_{12}\, V_{13} & n = 3 \\ V_{34}\, V_{23}\, V_{24}\, V_{12}\, V_{13}\, V_{14} & n = 4 \end{cases} \tag{1}$$

Here V_{ij} are the two-dimensional unitary matrices that mix the i-th and j-th generations. We see that an arbitrary unitary matrix allows such a representation. The nonphysical phases, which reduce to a redefinition of the quark-field phases, may be singled out by using a two-dimensional unitary matrix V_{ij} in one of the following forms:

$$V_{ij} = I_i \mathring{V}_{ij} I_i I_j , \quad V_{ij} = I_j \mathring{V}_{ij} I_i I_j , \quad V_{ij} = I_i I_j \mathring{V}_{ij} I_i , \quad V_{ij} = I_i I_j \mathring{V}_{ij} I_j , \tag{2}$$

where $\mathring{V}_{ij}$ is a two-dimensional orthogonal matrix, and I are the diagonal phase matrices:

$$\mathring{V}_{ij}=\begin{pmatrix} 1 & \cdots & & \cdots & 0 \\ \vdots & \ddots & & & \vdots \\ \cdot & c_{ij} & \cdots & -s_{ij} & \cdot \\ \vdots & \vdots & \ddots & \vdots & \vdots \\ \cdot & s_{ij} & \cdots & c_{ij} & \cdot \\ \vdots & & & \ddots & \vdots \\ 0 & \cdots & & \cdots & 1 \end{pmatrix}\begin{matrix} \\ \\ i \\ \\ j \\ \\ \\ \end{matrix},\quad I_j=\begin{pmatrix} 1 & & & 0 \\ & \ddots & & \\ & & e^{i\alpha} & \\ & & & 1 \\ 0 & & & & \ddots \end{pmatrix} j,\quad \begin{matrix} c_{ij}=\cos\theta_{ij}\geqslant 0 \\ s_{ij}=\sin\theta_{ij}\geqslant 0 \end{matrix}. \tag{3}$$

We can use the following parametrizations of V for three and four generations of quarks:

$$\begin{aligned} V &= \mathring{V}_{23}\, I_3(\alpha)\, \mathring{V}_{12}\mathring{V}_{13} & (n=3) \\ V &= \mathring{V}_{34}\, I_4(\gamma)\, \mathring{V}_{23}\, \mathring{V}_{24}\, I_4(\beta) I_3(\alpha)\, \mathring{V}_{12}\mathring{V}_{13}\, \mathring{V}_{14} & (n=4)\,. \end{aligned} \tag{4}$$

This choice has the following advantages: (a) The matrix elements of the first row, including $V_{ud}=V_{11}$ and $V_{us}=V_{12}$, are real. This circumstance simplifies the analysis of the CP violation in the K_L^0–K_S^0 system, since the amplitude of the direct decay of K^0 through the $(\bar{s}u)(\bar{u}d)$ interaction does not contain CP-odd phases in this case. (b) The available experimental data were obtained primarily for V_{ud}, V_{us}, V_{ub}, and V_{cb}, which immediately gives us, as will be shown below, the values s_{12}, s_{13}, and s_{23}, rather than a combination of angles, as in the parametrization of the Kobayashi-Maskawa model. (c) In the case of four generations, the parameters of the CP violation in K^0 mesons ϵ and ϵ', are determined by only two of the three phases if the mixing angles are small.

The explicit form of the V matrix can easily be derived from (4). We do not show it here because of the economy of space.

2. The available data on the measurement of the quark-mixing parameters[2] lead to the following restrictions: (a) s_{12}, s_{13}, s_{14}, and $s_{23}\ll 1$. (b) $s_{12}=0.221\pm 0.002$; (c) $s_{13}<0.005$ (Ref. 1); (d) $s_{14}<0.085$; (e) $s_{23}\simeq 0.044\pm 0.006$ (Ref. 1). (f) The constraint $|V_{cs}|>0.9$ (Ref. 4) leads to the condition $s_{24}<0.4$. The angle θ_{34} may be arbitrary [the indirect restrictions from $\Gamma(K_L\to\mu^+\mu^-)$, $\Delta m_{K_L K_S}$, and $\Delta m_{B^0\bar{B}^0}/\Gamma_{B^0}$ actually do not affect θ_{34}]. At first glance, the unnatural possibility $\theta_{24}\sim\theta_{34}\sim\theta_{12}\simeq 0.22$ can be interpreted as suggesting that the b and t quarks do, in fact, belong to the fourth generation and that the heavier b' and t' quarks belong to the third generation. In this case, we would have approximately the same mixing, $\sim\sin\theta_c$, of all successive generations, but the mixing occurring every other generation would be $\sim\sin^2\theta_c$.

3. The expression for the parameters of the CP violation in the decay of K^0 mesons, ϵ and ϵ', is[2]

$$\epsilon \simeq -\frac{e^{i\pi/4}}{2\sqrt{2}}\, z\left(\frac{\mathrm{Im}\, M_{box}}{\mathrm{Re}\, M_{box}} - 2\,\xi\right). \tag{5}$$

Here M_{box} is the amplitude of the transition $K^0 \to \bar{K}^0$ which is described by the "rectangular" diagrams, $\xi = \mathrm{Im}A_0/\mathrm{Re}A_0$, A_0 is the amplitude of the direct decay $K^0 \to 2\pi$ $(I = 0)$ from which the pion-scattering phase was separated and $z = \mathrm{Re}M_{box}/\mathrm{Re}M_{K\bar{K}} = 2\mathrm{Re}M_{box}/\Delta m_K$. For all $s_{ij} \ll 1$ we have

$$\frac{\mathrm{Im}\, M_{box}}{\mathrm{Re}\, M_{box}} = \frac{2 s_{13} s_{23}}{s_{12}} \sin\alpha \left[\kappa \ln \frac{m_t^2}{m_c^2} - 1 + \lambda \frac{s_{13} s_{23}}{s_{12}} \cos\alpha \frac{m_t^2}{m_c^2}\right]$$

$$+ \frac{2 s_{14} s_{24}}{s_{12}} \sin\beta \left[\kappa' \ln \frac{m_{t'}^2}{m_c^2} - 1 + \lambda \frac{s_{14} s_{24}}{s_{12}} \cos\beta \frac{m_{t'}^2}{m_c^2}\right]$$

$$+ \frac{s_{13} s_{23} s_{14} s_{24}}{s_{12}^2} \sin(\alpha + \beta)\, \rho\, \frac{m_t^2 m_{t'}^2}{m_c^2 (m_{t'}^2 - m_t^2)} \ln \frac{m_{t'}^2}{m_t^2}, \tag{6}$$

where the factors κ, λ, κ', λ', and ρ take into account the gluon corrections and the W-boson propagators. At $m_t = 40$ GeV and $m_{t'} = 60$ GeV, for example, we have $\kappa \simeq \kappa' \simeq 0.5$ and $\lambda \approx \lambda' \approx \rho \approx 0.7$–$0.8$.

The CP-odd phase for the direct decay $K^0 \to 2\pi$ $(I = 0)$ is

$$\xi = \frac{s_{13} s_{23}}{s_{12}} \sin\alpha H + \frac{s_{14} s_{24}}{s_{12}} \sin\beta H', \quad H \simeq H' \sim 1. \tag{7}$$

We can write the ratio ϵ'/ϵ as

$$\epsilon'/\epsilon \simeq \frac{\omega}{z} \cdot \frac{2\xi}{\dfrac{\mathrm{Im}\, M_{box}}{\mathrm{Re}\, M_{box}} - 2\xi} \exp i\left[(\delta_2 - \delta_0) + \pi/4\right], \quad \delta_2 - \delta_0 \simeq -\pi/4, \tag{8}$$

where $\omega = |A_2/A_0| \simeq 1/20$, $A_{0,2}$ are the amplitudes of the $K^0 \to 2\pi$ decays, and $\delta_{0,2}$ are the $\pi\pi$-scattering phases.

Expressions (5)–(8) show that if the last term in (6) is omitted, the third and fourth generations will contribute additively both to ϵ and to ϵ'.[1)] For this reason, if the fourth generation is mixed in with the first two generations much less vigorously than the third generation ($s_{14}s_{24} \ll s_{13}s_{23}$), the values of the Kobayashi-Maskawa model predicted for ϵ and ϵ'/ϵ will remain essentially the same. If $s_{14}s_{24} \sim s_{13}s_{23}$, however, the value of ξ, and hence the value of ϵ'/ϵ will decrease appreciably and may even change sign. Since $H \simeq H'$, ϵ' will vanish only if $s_{13}s_{23}\sin\alpha$ is approximately equal to $-s_{14}s_{24}\sin\beta$, which substantially decreases the value of ϵ in the logarithmic terms that are proportional to $s_{13}s_{23}/s_{12}$ and $s_{14}s_{24}/s_{12}$. However, since the terms quadratic in m_t and $m_{t'}$ may be of the same order of magnitude as the logarithmic terms, the absolute value of ϵ may well remain the same.

The experimental value of ϵ and the ratio ϵ'/ϵ of either sign can thus account for the existence of the fourth generation only if the mixing matrix does not have a hierarchical structure when the fourth generation is mixed in with the first two generations not nearly as strongly as the third generation.

4. Although the fourth generation can substantially change the mass difference and the *CP*-odd mixing $D^0 \leftrightarrow \overline{D}^0$ and $B^0 \leftrightarrow \overline{B}^0$, these effects still lie outside the range of experimental observation. In the case of B_s^0–$\overline{B}_s^0$ system, in which $\Delta m_{B_s}/\Gamma_{B_s} \simeq 1.5$ (at $m_t \simeq 40$ GeV) in the standard model is determined solely by the mass of the t quark and $\epsilon_{B_s} \sim 10^{-3}$, the fourth generation can easily lead to both an appreciable change in $\Delta m_{B_s}/\Gamma_{B_s}$ (in particular, $\Delta m_{B_s} \gg \Gamma_{B_s}$) and a marked increase in ϵ_{B_s} due to the presence of the γ phase in the mixing of t and t' quarks.

[1)]Expressions (6) and (7) are valid only if $s_{24}s_{14} \sim s_{23}s_{13}$. If $s_{24}s_{34} \gtrsim s_{23}$ or $s_{14}s_{34} \gtrsim s_{13}$, additional terms containing the γ phase will appear in (6) and (7). The contribution of the fourth generation to Δm_K is omitted in (6).

[1]J. Lee-Franzini, Talk at the XXII International Conference on High Energy Physics, Leipzig, July 1984.

[2]A. J. Buras, W. Slominski, and H. Steger, Nucl. Phys. B **238**, 529 (1984); N. G. Ural'tsev and V. A. Khoze, Proceedings of the XIX Winter School, Institute of Nuclear Physics, Leningrad, 1984, p. 3; Usp. Fiz. Nauk **146** (1985) (in press).

[3]K. Nishikawa, Talk at the XXII International Conference on High Energy Physics, Leipzig, July, 1984.

[4]T. M. Aliev, V. L. Eletskiĭ, and Ya. I. Kogan, Yad. Fiz. **40**, 823 (1984) [Sov. J. Nucl. Phys. **40**, 527 (1984)]

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