Electromagnetic form factors of pseudoscalar mesons at low momentum transfer in quantum chromodynamics

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The finite-energy sum-rule technique and the generalized operator expansion are used to determine the behavior of the electromagnetic form factors of π^+, K^+, and K^0 mesons at small momentum transfer $\lesssim 0.7\ \mathrm{GeV}^2$ in quantum chromodynamics.

Considerable progress has recently been made in extending the range of applicability of both the finite-energy QCD sum rules[1] and the QCD sum rules based on the use of the Borel transformation.[2] A generalization of the original method proposed by several authors[3] has made it possible to study the three-point Green's functions and the hadron characteristics associated with these functions, in particular, the electro-

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magnetic form factor of the π meson in the momentum-transfer range 0.7 $GeV^2 \lesssim Q^2 \lesssim 4$ GeV^2. Here the lower bound stems from the fact that the standard operator expansion, which is routinely used in the sum-rule technique, is inapplicable at small values of Q^2. It was shown in Refs. 4 and 5 that this difficulty can be overcome by using the so-called generalized operator expansion[5] (or, equivalently, the standard operator expansion in a slightly varying external field[6]), which makes it possible to study the three-point Green's functions in a region in which only one external momentum is large. A characteristic feature of the generalized operator expansion are the new phenomenological quantities: the vacuum-expectation values of the bilocal operators which parametrize, along with the condensates of the local operators, the nonperturbative corrections in the corresponding power series expansions.

In the present letter we use the generalized operator expansion and the finite-energy QCD sum-rule technique[1,7] to calculate the electromagnetic form factors of the π^+, K^+, and K^0 mesons in the momentum-transfer range $0 \leqslant Q^2 \leqslant 0.7$ GeV^2. We should point out that the form factor $F_\pi(0 \leqslant Q^2 \lesssim 0.8$ $GeV^2)$ was studied in Refs. 5 and 8 within the context of the generalized operator expansion and the Borel sum rules. It is of particular interest to calculate the electromagnetic form factors of the pseudoscalar mesons that contain the s quark by using the QCD sum-rule technique for the following reasons. First, the absence of relevant experimental data (with the exception of the mean-square radii) allows us to test the predicting capability of the sum-rule technique and, secondly, in the case of the K^0 meson the total effect is attributable to the SU(3) symmetry violation, which accounts for the high sensitivity of the results to the choice of the parameters responsible for the violation of this symmetry.

Let us examine a Green's function of the type

$$T_{\mu\alpha\beta}(p,q) = i^2 \int dx dy\, e^{ipx - iqy} \langle T(j_\alpha(x) J_\mu(y) j_\beta^+(0)) \rangle_0 \tag{1}$$

in the region of large euclidean p^2 and small euclidean q^2. J_μ is the electromagnetic hadronic current, and j_α is the axial current which has a nonzero projection on the corresponding single-meson state (for a K^+ meson, for example),

$$j_\alpha = \bar{s}\gamma_\alpha\gamma_5 u \quad \text{and} \quad \langle 0 | j_\alpha(0) | K^+(p) \rangle = i f_K p_\alpha .$$

Phenomenologically, the contribution of a meson (π^+, K^+, or K^0) to $T_{\mu\alpha\beta}(p,q)$ is proportional to $p_\alpha(p-q)_\beta(2p-q)_\mu f^2 F(q^2)$, where f is the decay constant (f_π for π^+, f_K for K^+ and K^0). There are thus four structures ($p_\alpha p_\beta p_\mu$, $p_\alpha q_\beta q_\mu$, $p_\alpha q_\beta p_\mu$, $p_\alpha p_\beta q_\mu$) which can be used to calculate the form factor $F(q^2)$. We will use here the second structure. The corresponding invariant amplitude $T(p^2,q^2,pq)$ satisfies a double dispersion relation in terms of the variables p^2 and $(p-q)^2$. This dispersion relation gives us a phenomenological representation for T. We can obtain a theoretical representation for T by using the generalized operator expansion for the amplitude $T_{\mu\alpha\beta}$:

$$T_{\mu\alpha\beta}(p,q)_{p^2 \to -\infty} = \sum_i K^i_{\mu\alpha\beta}(p,q) \langle O_i(0) \rangle_0 + \sum_j C^j_{\alpha\beta}(p) \langle i \int dy\, e^{-iqy} T(J_\mu(y) O_j(0)) \rangle_0 , \tag{2}$$

where $\{O_i\}$ is the complete set of local operators. Since the coefficient functions K^i

and C^j are determined by the dynamics of the theory at short ranges ($\sim 1/|p|$), they can be accurately calculated within the context of the perturbation theory. Further calculations can be simplified by assuming that $p^2 = (p-q)^2$. This simplification allows us to express all the momentum variables in the expansion in terms of two parameters: $t = -p^2$ and $Q^2 = -q^2$.

Working from the generalized operator expansion for the amplitude T (for a π^+ meson, for example, we have

$$T^{\text{theo}}(t, Q^2) = \frac{1}{t}\left\{ \frac{-1}{12\pi^2} + \frac{1}{4\pi^2}\ln\left(\frac{\mu^2}{t}\right) - \Pi_1^{(u)}(Q^2)\right\},$$

$$-\frac{Q^2}{t^2}\left\{\frac{1}{16\pi^2} - \frac{1}{8\pi^2}\ln\left(\frac{\mu^2}{t}\right) + \Pi_2^{(u)}(Q^2)\right\} + \frac{1}{t^2}\Pi_3^{(u)}(Q^2) + O(t^{-3}), \qquad (3)$$

the expansions for K^0 and K^+ mesons are similar), we find the corresponding spectral density, from which we can obtain, by using the finite-energy sum rules, information on the phenomenological spectral density in the dispersion relation for $T(t,Q^2)$, and hence information on the form factor of interest to us. In (3) the functions $\Pi_i(Q^2)$, denote the bilocal-operator contributions. The function $\Pi_2(Q^2)$, for example, can be expressed as

$$i\int dy e^{-iqy} \langle T(\bar\psi(y)\gamma_\mu \psi(y)\bar\psi(0)\gamma_\beta \hat p(i\hat D)\hat p\gamma_\alpha\psi(0))\rangle_0$$

$$= p_\alpha q_\beta q_\mu\, 2(pq)\,\Pi_2^{(\psi)}(-q^2) + \text{other structures}, \qquad \psi = u, d, s.$$

The procedure for calculating the bilocal contributions is outlined in Refs. 4 and 9. Using this procedure for $\Pi_2^{(u)}(Q^2)$, we can write

$$\Pi_2^{(u)}(Q^2) = -\frac{1}{24\pi^2} + \frac{\langle \alpha_s G^2\rangle_0}{24\pi\mu^4} + \frac{s_1}{8\pi^2}\left\{\frac{1}{m_\rho^2 + Q^2} - \frac{1}{m_\rho^2 + \mu^2}\right\}$$

$$+\frac{1}{8\pi^2}\ln\frac{(s_1+\mu^2)}{(s_1+Q^2)}, \qquad s_1 = 1.24\,\text{GeV}^2. \qquad (4)$$

We finally find the following equation for $F_{K^+}(Q^2)$:

$$F_{K^+}(Q^2) = e_u\left\{\frac{s_0}{4\pi^2} - \frac{(M_K^2 + 3m_s^2 + 15m_u^2)}{12\pi^2} + \frac{1}{4\pi^2}[M_K^2 - m_s^2 + m_u^2]\ln\left(\frac{\mu^2}{s_0}\right)\right.$$

$$+ (m_s^2 - M_K^2)\Pi_1^{(u)}(Q^2) + \Pi_3^{(u)}(Q^2) + m_s\Pi_4^{(u)}(Q^2) - Q^2\left[\frac{3}{16\pi^2} - \frac{1}{8\pi^2}\ln(\mu^2/s_0)\right.$$

$$\left.\left. + \Pi_2^{(u)}(Q^2)\right]\right\} - e_s\{u \leftrightarrow s\} \qquad (5)$$

Here e_i and m_i are the charge and mass of the i-th quark, and M_K is the mass of the K meson. The form factors of the K^0 and π^+ mesons can be determined from (5) by

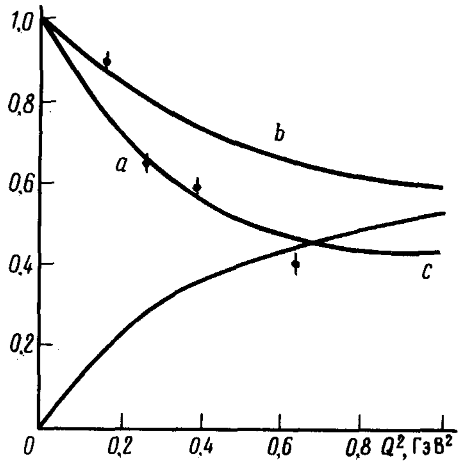

FIG. 1. Curves a, b, and c are respectively the calculated values of the form factors $F_\pi(Q^2)$, $F_{K^+}(Q^2)$, and $10\times F_{K^0}(Q^2)$. The experimental data, taken from Ref. 14, refer to the form factor of the π^+ meson.

substituting $(u \to d)$ and $(s \to u, M_K \to 0)$, respectively. The value of the parameter s_q (the continuum threshold) was chosen with the stipulation that the form factor be normalized to allow for the charge of the corresponding meson with $Q^2 = 0$. For π^+, for example, we find $s_0 = 0.7\ \mathrm{GeV}^2$ if $F_{\pi^+}(0) = 1$. This value is approximately equal to the threshold given by the duality equation: $s_0 = 1/2(M_\pi^2 + M_{A_1}^2) = 0.8\ \mathrm{GeV}^2$. The parameter μ in Eqs. (4) and (5) corresponds to the normalization point in the $\overline{MS}$ scheme. We assume that $\mu = 1$ GeV and m_s (1 GeV) = 200 MeV (Ref. 10); standard values were chosen for the parameters $\langle\bar{\psi}\psi\rangle_0$ and $\langle\alpha_s G^2\rangle_0$ appearing in the bilocal contributions.[2] As expected, after incorporating the μ dependence of the bilocal contributions, the final equations for the form factors will no longer depend on μ (provided that μ is not too small).

The plots of the form factors calculated by us $F_\pi(Q^2)$, $F_{K^+}(Q^2)$, and $F_{K^0}(Q^2)$ are shown in Fig. 1. We should keep in mind that the actual parameter of the expansion in Eq. (4) is $x = Q^2/s_0$, so that the range of applicability of these results is bounded above by the condition $Q^2 \lesssim s_0$. The use of Eq. (4) allows us to calculate the mean square charge radii of π^+, K^+, and K^0: $\langle r_\pi^2\rangle = 0.46\ \mathrm{fm}^2$, $\langle r_{K^+}^2\rangle = 0.25\ \mathrm{fm}^2$, and $\langle r_{K^0}^2\rangle = -0.042\ \mathrm{fm}^2$.

A few words about the accuracy of our results are in order. Strictly speaking, the accuracy of these results depends on both the accuracy of the method used to determine the bilocal operators and the higher-order power-law corrections to the T^{theor} function. An estimate of the bilocal-operator contribution, $\sim 1/t^3$, shows that it lies well within several percent. As regards the higher-order local corrections, they are at least partially accounted for by appropriately choosing the parameter s_0 (by normalizing the form factor to allow for the charge, with $Q^2 = 0$). We thus conclude that the error in our results stems primarily from the use of the QCD sum rules to determine the bilocal operators for small values of Q^2, and hence should be no greater than 15–20%. We easily see that our values of the mean square radii are, within this error margin, in agreement with the experimental data of Refs. 11–13:

$$< r_{\pi}^2 > = 0.430 \pm 0.014\,\mathrm{fm}^2, \qquad < r_{K^+}^2 > = 0.28 \pm 0.05\,\mathrm{fm}^2,$$

$$< r_{K^0}^2 > = -\,0.054 \pm 0.026\,\mathrm{fm}^2.$$

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