Hopping conductivity of germanium bicrystals in strong electric fields at $T < 1$ K

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The electrical conductivity $\sigma_{\square}$ of germanium bicrystals with a tilt angle $\theta \approx 8.5°$ is studied as a function of the strength of the electric field E in the region near the hopping conductivity over the temperature range $0.05 \leqslant T \leqslant 1$ K. As the electric field is increased, the region $\sigma(E) \approx \text{const}$ gives way to an interval of fields, in which $\sigma \sim \exp \gamma E$ and the coefficient γ increases with decreasing T. At the lowest temperatures, $T \leqslant 0.1$ K, the functional dependence $J \sim \exp[-(E_0/E)^{1/2}]$ is found to be consistent with the theoretical prediction of Efros and Shklovskiĭ.

Semimetallic conduction in germanium bicrystals breaks down abruptly at low temperatures.[1] As soon as the conduction (under the influence of a logarithmic decrease in electrical conductivity[2,3]) reaches a value corresponding to the minimum metallic conduction in a two-dimensional medium,[4] $\sigma_{\min} \approx e^2/h$, the electrical conduc-

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Table I.

Nº	$\sigma_{\square}$, Ω^{-1}	p, cm^{-2}	σ_0, Ω^{-1}	T_0, K
1	5.5×10^{-5}	4.1×10^{12}	7×10^{-4}	11.5
2	4.0×10^{-5}	4.0×10^{12}	6×10^{-4}	20

p and $\sigma_{\square}$ are the hole density and the electrical conductivity at $T=4.2$ K.

tivity changes sharply and with further decrease in T the conduction decreases in accordance with the exponential function

$$\sigma(T)=\sigma_0 \exp[-(T_0/T)^{1/2}], \tag{1}$$

where e is the electronic charge, h is Planck's constant, $\sigma_0 \approx$const, and the parameter T_0 depends on the properties of the sample.

These systematic features were determined in an experiment in which the current depended linearly on the applied voltage. To complement an earlier experiment,[1] we have carried out measurements in strong electric fields which enabled us to determine the mechanism for the conductivity of Ge bicrystals at $\sigma_{\square} < \sigma_{\min}$.

We used Ge bicrystals with a slope angle $\theta \approx 8.5°$ and electrical conductivity $\sigma_{\square}$ approximately equal to $\sigma_{\min}$ to study $\sigma = f(T,E)$ at $T<1$ K. The characteristics of the samples being studied are listed in Table I.

The samples tested exhibited a linear relationship between the field E and the current J over a narrow interval of electric fields which decreased with decreasing temperature: At $T=1$ K it reached about 1 V/cm and at $T=0.3$ K it was within several mV/cm. Under these conditions, the region $\sigma \cong$ const comprised only a small part of the range of fields studied, $0 \leqslant E \leqslant 3$ V/cm.

To identify this region and to determine the systematic features of the transition to "strong" fields, we have represented the measurement results in Figs. 1 and 2 as a function $\ln\sigma = f(E)$. We see from these data that at a certain strength of the field E_1, Ohm's law gives way to an exponential increase of the conductivity

$$\sigma(T,E)=\sigma(T)\exp[\gamma(E-E_1)], \tag{2}$$

where $\gamma = \Delta \ln \sigma/\Delta E$ is a constant which characterizes the slope of the curves. The quantity γ increases sharply with decreasing T, and the range (2) over which this change occurs diminishes.

In reasonably strong electric fields, the curves $\sigma = f(T,E)$ become progressively flatter and finally combine into a single smooth curve which does not depend on T, as we can see from the data in Fig. 2.

In the region of lowest temperatures, $0.05 \leqslant T \leqslant 0.10$ K, the density of the current J (in fields $E>1$ mV/cm) depends solely on the electric-field strength. In this temperature range, the measurement results for $J = f(T,E)$, as we can see from the data in Fig.

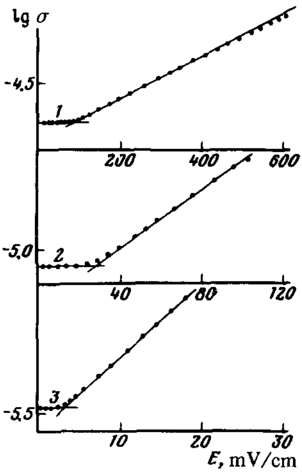

FIG. 1. Electrical conductivity versus the electric field of sample No. 1 at the following temperatures: 1—1 K; 2—0.6 K; 3—0.4 K.

3, are approximated well by the expression

$$J \sim \exp\left[-(E_0/E)^{1/2} \right], \qquad (3)$$

where E_0 is a constant that depends on the properties of the sample. The values of E_0 determined from the slope of the experimental curves are given in Table II.

At conductivities $\sigma_\square < \sigma_{\min}$, the conduction is realized by hopping from one localized state to another which lags a distance R, a distance greater than the localization

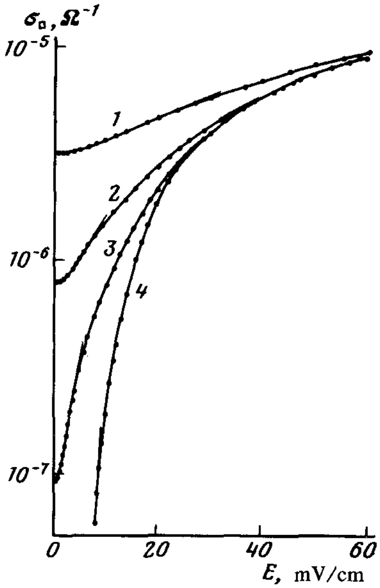

FIG. 2. Electrical conductivity σ versus the electric field E of sample No. 1 at the following temperatures: 1—0.4 K; 2—0.25 K; 3—0.15 K; 4—0.05–0.10 K.

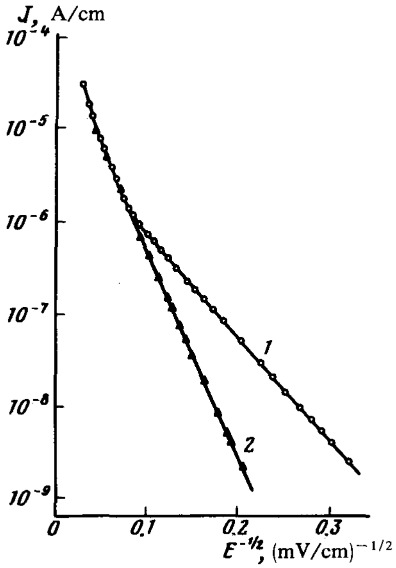

FIG. 3. Current density J versus the inverse field strength, $1/\sqrt{E}$, of samples No. 1 and No. 2. 1—Sample No. 1; 2—sample No. 2.

length R_L, behind the first state. The probability for such hopping is proportional to

$$\exp(-R/R_L)\exp(-W_a/kT), \tag{4}$$

where k is the Boltzmann constant, and W_a is the activation energy.

If the Coulomb interaction is dominant,[6,7] we see that the activation energy $W_a \simeq e^2/\kappa R$ (where κ is the dielectric constant). Optimization of expression (4) gives us exponential function (1), in good agreement with the measurement results for bicrystals in weak electric fields.[1] The values of T_0 determined from the slopes of the experimental curves, $\sigma = f(1/\sqrt{T})$, are given in Table II, along with the localization lengths R_L, which were calculated from the following expression, using the procedure obtained in Ref. 5:

$$R_L = 4e^2/\kappa k T_0. \tag{5}$$

Upon application of an electric field of adequate strength, the activation energy

$$W_a = e^2/\kappa R - eER \tag{6}$$

Table II.

N°	T_0, K	E_0, V/cm	Localization length, cm		L_m, cm
			From (5)	From (10)	
1	11,5	0,7	$3{,}6\times10^{-5}$	10×10^{-5}	$>10^{-5}$
2	20	2,5	$2{,}1\times10^{-5}$	6×10^{-5}	$>10^{-5}$

decreases markedly. Optimization of expression (4) under these conditions gives us the functional dependence

$$\sigma(T, E) = \sigma(T)\exp(eER_m/kT) \tag{7}$$

for intermediate electric fields, in which the effect of the field E on the length of the "optimal" jumps, R_m, can still be ignored.[5] Expression (7) is in qualitative agreement with the measurement results in Fig. 1, although it predicts a slower increase in γ with decreasing T than that observed experimentally.

With further increase in the electric field, the activation energy tends toward zero, as is evident from expression (6). As a result, the jumps span a distance R_E given by

$$e^2/\kappa R_E - eER_E \cong 0. \tag{8}$$

Under these conditions the electron moves in the direction of the electric field, emitting phonons after each jump.[8,9] In this case the current is

$$J \sim \exp[-(E_0/E)^{1/2}], \tag{9}$$

and the localization length is related to the quantity E_0 by

$$R_L = \sqrt{e/\kappa E_0}, \tag{10}$$

where E_0 is the characteristic electric field.

Table II gives the experimental values of E_0 and the values of R_L calculated from (5) and (10). In calculating R_L in first approximation, we assumed that $\kappa = 16$ for both Ge bicrystals and single crystals.

We see from the data in Table II that the values of R_L determined from expressions (5) and (10) vary by no more than a factor of three. This is a reasonably good agreement, since the original equations for the two-dimensional case are accurate only within numerical factors.

The agreement between the values of R_L obtained from measurements under different experimental conditions conclusively supports the "Coulomb-gap" model[6,7] upon which the calculations of R_L were based.[10] Another argument in support of this model are the effects, called "Coulomb-gap signs" by Al'tshuler *et al.*,[3] which play a significant role in Ge bicrystals in the region of semimetallic conduction.[11,12]

It was shown in these studies that the logarithmic decrease in the conduction of Ge bicrystals is determined by the electron-electron interaction.[3] The electrical conductivity changes in this case with temperature in proportion to $\ln L_T$, where $L_T = \sqrt{D_\tau}$ is the coherence length, $\tau = \hbar/kT$ is the interaction time, and D is the diffusion coefficient which is related to the conductivity σ in a two-dimensional medium by

$$\sigma = (e^2 m/\pi\hbar^2) D; \tag{11}$$

here m is the effective mass of the carriers.

If a gradually decreasing conductivity $\sigma_\square$ approaches a threshold value $\sigma_{\min}$, the diffusion coefficient will approach a limiting value $D_{\min} \simeq \hbar/m$, and the coherence length

$$L_m \geqslant \sqrt{D_{\min}(\hbar/kT_c)} \approx \hbar/\sqrt{mkT_c} \tag{12}$$

will increase as the critical temperature T_c, at which the semimetallic conduction is disrupted, is reduced. In the samples studied by us, T_c is about 1 K, and hence the coherence length is

$$L_m \geqslant 10^{-5}\,\mathrm{cm}. \tag{13}$$

As we can see from the data in Table II, the value of L_m obtained by us is in reasonable order-of-magnitude agreement with the localization length R_L.

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