Echos in classical and quantum ensembles with determinate frequencies

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New coherent responses of the echo type arise in ensembles of a finite number of elements with specified frequencies. These responses are discussed theoretically from the quantum-mechanical standpoint. They have been observed experimentally in a classical situation.

1. We know that when an ensemble of oscillators with a spread in resonant frequencies are excited by two pulses separated by a time interval T, a coherent response (an echo) will appear at a time $2T$ after the first pulse. The echo method has become a general-purpose method in research on the physics of various entities for which the inhomogeneous line broadening in the ensemble exceeds the oscillator width.[1] The systems which have been studied have contained large numbers of oscillators, so that the distribution of their frequencies could be treated as continuous. Only recently has there been a study of a discrete system of oscillators.[2] Systems of this type are interesting for research on small numbers of particles, quantum oscillators, and the statistical properties of echos. In this letter we wish to call attention to some qualitatively new aspects of echos in quantum-mechanical and classical systems when there is a determinate set of frequencies.

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We know that when oscillators are excited by a pulse, a phase coherence will arise in the ensemble, along with the responses, after a time which is a multiple of $\tau = 2\pi/\Delta$, where Δ is the constant frequency step. Echos will appear not only at the time $2T$ but also at $t_n = 2T + n\tau$, where $n = 1, 2, \ldots$. Here we consider a quantum-mechanical system with determinate frequencies (a Rydberg atom), and we show that the echo and the train of pulses that it generates are accompanied by the appearance of some new responses. These new responses, of a different nature, are seen in a classical ensemble at the times $t_n = -T + n\tau$, where the integer n is greater than $2T/\tau$. We call these responses an "imaginary echo." We also report the observation of this effect here.

2. We consider a quantum-mechanical system with determinate frequencies. When an atom is acted upon by a short pulse ($\delta t \sim 10^{-11}$ s) with a frequency near the ionization threshold, a set of Rydberg levels is excited. The emission field of this atom is a superposition of the fields emitted as a result of transitions between Rydberg levels and the ground level. Their frequencies are different, and if the delay satisfies the condition $T \gg \delta t$, the necessary conditions for an echo will be met. The elements of the ensemble are concentrated in a single particle, and there is the possibility here of observing a collective effect (an echo) in a single atom. If the field frequency is so close to the ionization threshold that only a small part of the Rydberg spectrum is excited, and we can assume that the levels in this small part of the spectrum are uniformly spaced, then we have a quantum-mechanical ensemble in which we can also observe a periodic train of emission pulses. If the first pulse excites an atom to one term, the second pulse will invert the phase of the wave function of the atom,[3,4] but the atom will be in a different term and will emit at a different frequency. In other words, the linear oscillations of one element of the ensemble excite nonlinear oscillations of another; i.e., a coherent interaction is established among the elements of the ensemble. Since the corresponding frequencies are different, there is a change in the time evolution of the phase, while the phase shift remains constant. As a result, there is a change in the time at which the phase shift is cancelled, and this change gives rise to a new response. Analysis shows that the response occurs at all times T_0 such that[1)] $T_0/T = s/r$, where s and r are integers. The entire pattern of responses is repeated periodically at intervals of time τ. The intensity of the emission by a Rydberg atom localized near T_0

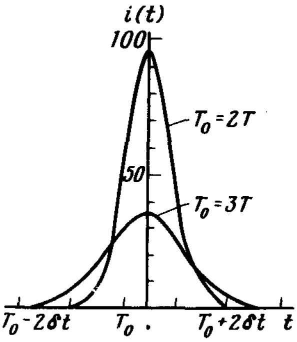

FIG. 1. Shape of the emission pulses from a Rydberg atom; the emission is localized in time at $T_0 = 2T$ and $T_0 = 3T$.

is described by

$$I(t)=\frac{4\,(\omega\,|d|^2 E\,\delta t/\hbar)^4 E^2}{3\,\hbar^2 c^3 \Delta^2}\,i(t);$$

$$i(t)=\Bigg\{\frac{\pi r}{4(s-r)^2}\Bigg[\frac{r\,(2s-r)}{s^2}\,\theta_2\Big(2\frac{s}{r}-1-|x|\Big)$$

$$-2\Big(1+\Big(\frac{s-r}{s}\Big)^2\Big)\theta_2(1-|x|)-\theta_2\Big(2\frac{s}{r}-3-|x|\Big)$$

$$+\Big(\frac{s-r}{s}\Big)^2\theta_2\Big(2\frac{s}{r}+1-|x|\Big)+\theta_0\Big(2\frac{s}{r}-1-x\Big)$$

$$\times\Big(\Big(1+\frac{r}{s}\Big)\theta_2(x-1)+\Big(1-\frac{r}{s}\Big)\Big(\theta_2\Big(x+2\frac{s}{r}-1\Big)$$

$$-\theta_2(x+1)\Big)-\theta_2\Big(x+2\frac{s}{r}-3\Big)\Big)\Bigg]\Bigg\}^2, \qquad (1)$$

where $x=2(t-T_0)/\delta t-1$; $\theta_k(x)=\{^{x^k}_0$ for $x \gtrless 0$; $2E$ and ω are the field amplitude and frequency; and d is the dipole matrix element of the transition between the ground and Rydberg levels. This expression was derived under the assumption that the pulses are square and weak, that d remains constant over the excited part of the spectrum, and that the condition $\Delta \ll 1/\delta t$ holds. Figure 1 shows the shape of the pulses described by (1).

3. An echo has been observed in a system of classical nonlinear oscillators: an ensemble of pendulums. The apparatus is described in Ref. 2. In contrast with the experiments of Ref. 2, the oscillation frequencies of the 15 pendulums were spaced uniformly at an interval $\Delta = 0.016$ Hz ($\nu_k = 1.24$ Hz $+ k\Delta$, where $k = 1, 2, \ldots, 15$), within an error $\sim 10^{-3}$ Hz. Features attributable to the uniform spacing arise even in the case of excitation by a single pulse (Fig. 2a). We see that the response localized at the initial time generates a train of pulses separated by time intervals $\tau = 2\pi/\Delta = 60$ s. These pulses result from a coupling of the linear oscillations, which is easy to observe visually. After the second pulse, a train of responses appears at times $T + n\tau$. As the intensity of the pulses is raised, the echo signals appear at the times $2T + n\tau$ and $-T + n\tau$ (Fig. 2, c and d). After the two pulses, the linear oscillation is a superposition of two oscillations with phases $\varphi_l^{(1)} = \nu t$ and $\varphi_l^{(2)} = \nu(t-T)$. These phases are added because of the nonlinearity. If the nonlinearity is cubic, there can be two combinations of phases: $\varphi_{nl}^{(1)} = 2\varphi_l^{(2)} - \varphi_l^{(1)} = \nu(t-2T)$ and $\varphi_{nl}^{(2)} = 2\varphi_l^{(1)} - \varphi_l^{(2)} = \nu(t+T)$. They correspond to a phase coherence of the oscillations of oscillators in the future, at $t = 2T$ (the echo) and in the past, at $t = -T$ (the imaginary echo). Because of the uniform level spacing, there will also be a phase coherence at the times $t_n = 2T + n\tau$ and $t_n = -T + n\tau$, making it possible to observe the imaginary echo. The relative

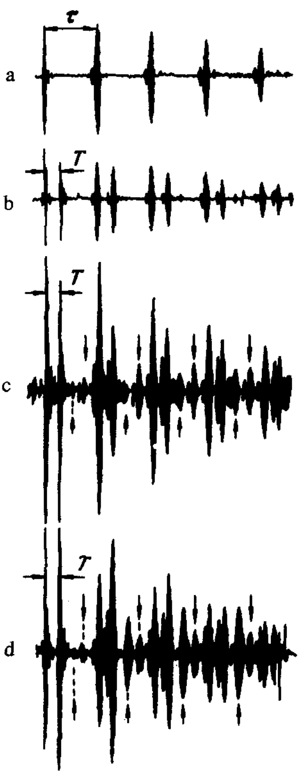

FIG. 2. a—Linear response of an ensemble of pendulums to a single excitation pulse; b—the same, but for two excitation pulses, $T = 20$ s; c—linear and nonlinear responses of the ensemble to two pulses ($T = 15$ s), the first of which is the more intense; d—the same, but the second pulse is the more intense. ↑) Echo responses; ↓) imaginary-echo responses.

intensities of the ordinary and imaginary echos depend on the relative intensities of the excitation pulses, as is confirmed experimentally.

An interesting question is the minimum number of oscillators required for an echo. The best ensemble here is one with a uniformly spaced spectrum and with an inhomogeneous width less than $(\delta t)^{-1}$. Under such conditions we have observed an echo from five oscillators.

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[1)]Similar responses occur[5] in a gas excited by standing-wave pulses. In that case, however, the responses stem from a pronounced modulation of the polarization of the gas.

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