Axial anomaly in supersymmetry gauge theories

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An explicit relationship between the axial-current operator, which satisfies the Adler-Bardeen theorem, and the axial supersymmetric current, which appears in the supercurrent, is found. The axial and superconformal anomalies are mutually consistent in all orders of perturbation theory.

1. The axial anomaly in supersymmetry gauge theories has recently drawn close attention of many investigators.[1–6] This anomaly can be summarized as follows. According to the Adler-Bardeen theorem, the divergence of the axial current contains only the first order in α (Ref. 7)

$$\partial_\mu j^5_\mu = -\frac{\alpha N}{4\pi}\mathcal{F}_{\mu\nu}\tilde{\mathcal{F}}_{\mu\nu} \tag{1}$$

[N refers to the SU(N) gauge group], whereas the superconformal anomaly, which also contains the axial anomaly, is proportional to the β function, i.e., it includes all orders of perturbation theory.[8]

$$\partial_\mu J^5_\mu = \frac{1}{3}\frac{B(\alpha)}{\alpha}\left[F_{\mu\nu}\tilde{F}_{\mu\nu} + 2\partial_\mu J^5_\mu\right]. \tag{2}$$

The other contradiction is the anomalous dimension of the current. For a current $A - Bj^5_\mu$, the anomalous dimension is nonvanishing. At the same time, the axial current J^5_μ is a component of the supermultiplet which includes the energy-momentum tensor which evidently has a zero anomalous dimension. This situation is incompatible with the assumption that an anomaly produces a supermultiplet.

The solution of this problem can be sought in the renormalizations. Discrepancies can be resolved if we take into account that we are dealing with two different renormalization schemes. The AB theorem is encountered in a special scheme which breaks the supersymmetry. The quantum operators in the supersymmetry scheme are not the same as those in the AB scheme, but they are connected by finite multiplicative transformations. This circumstance applies not only to the currents but also to the $F\tilde{F}$ operators in the axial anomalies (1) and (2). Renormalization of the currents alone, which is insufficient as will be shown below, has generated several contradictory assertions in the literature.[1–4]

2. Let us examine the renormalization operators in the axial anomalies (1) and (2). These operators satisfy the standard renormalization-group equations which are written in the corresponding renormalization scheme:

$$\left[\mu\frac{\partial}{\partial\mu} + \beta(a)\frac{\partial}{\partial a} - \hat{\gamma}(a)\right]\begin{pmatrix}\partial_\mu j^5_\mu(a)\\ \mathcal{F}_{\mu\nu}\tilde{\mathcal{F}}_{\mu\nu}(a)\end{pmatrix} = 0, \tag{3}$$

$$[\mu \frac{\partial}{\partial\mu} + B(A)\frac{\partial}{\partial A} - \hat{\Gamma}(A)] \begin{pmatrix} \partial_\mu J^5_\mu (A) \\ F_{\mu\nu}\widetilde{F}_{\mu\nu}(A) \end{pmatrix} = 0, \tag{4}$$

where $\beta(a)$ and $B(A)$ are the β functions, $a \equiv \alpha_{AB} N/2\pi$, $A \equiv \alpha_{CC} N/2\pi$ are the renormalization charges, and the matrices for the anomalous dimensions are triangular in shape[6,9]:

$$\hat{\gamma} = \begin{pmatrix} \gamma_{11} & 0 \\ \gamma_{21} & \gamma_{22} \end{pmatrix}, \quad \hat{\Gamma} = \begin{pmatrix} \Gamma_{11} & 0 \\ \Gamma_{21} & \Gamma_{22} \end{pmatrix}. \tag{5}$$

These matrices can be found by direct calculation, but certain general properties can be inferred from the renormalization-invariant anomalies[9]:

$$[\mu \frac{\partial}{\partial\mu} + \beta(a)\frac{\partial}{\partial a}] \{ \partial_\mu j^5_\mu + \frac{a}{2} \mathcal{F}_{\mu\nu} \widetilde{\mathcal{F}}_{\mu\nu} \} = 0, \tag{6}$$

$$[\mu \frac{\partial}{\partial\mu} + B(A)\frac{\partial}{\partial A}] \{ \partial_\mu J^5_\mu - \frac{B(A)}{3A} [F_{\mu\nu}\widetilde{F}_{\mu\nu} + 2\partial_\mu J^5_\mu] \} = 0. \tag{7}$$

Combining Eqs. (3), (6), (4), and (7), we can write the matrices in (5) in the form

$$\hat{\gamma} = \begin{pmatrix} \gamma_{11} & 0 \\ -\frac{2\gamma_{11}}{a} & -\frac{\beta(a)}{a} \end{pmatrix}, \quad \hat{\Gamma} = \begin{pmatrix} 0 & 0 \\ -2A\left(\frac{B}{A}\right)' & -A\left(\frac{B}{A}\right)' \end{pmatrix}. \tag{8}$$

In these matrices we have allowed for the fact that the anomalous dimension of the current is $\Gamma_{11} = 0$ in the CC scheme. In the AB scheme, the anomalous dimension of the current vanishes in single-loop approximation; it then becomes nonvanishing[2,9]:

$$\gamma_{11}(a) = -\frac{3}{2} a^2 + O(a^3).$$

3. Working from the general propositions of the renormalization theory, we conclude that the renormalized operators of the various schemes are related to each other by a finite multiplicative transformation:

$$\begin{pmatrix} \partial_\mu J^5_\mu (A) \\ F_{\mu\nu}\widetilde{F}_{\mu\nu}(A) \end{pmatrix} = \begin{pmatrix} S_{11}(A) & 0 \\ S_{21}(A) & S_{22}(A) \end{pmatrix} \begin{pmatrix} \partial_\mu j^5_\mu (a) \\ \mathcal{F}_{\mu\nu}\widetilde{\mathcal{F}}_{\mu\nu}(a) \end{pmatrix}, \tag{9}$$

where $A = Z(A)a$.

Using Eq. (9), we can write the condition under which Eqs. (1) and (2) can be combined as

$$B(A)=-\frac{3A^2}{2}\frac{S_{11}}{S_{22}Z}\left(1-A\frac{S_{11}+\frac{1}{2}S_{21}}{S_{22}Z}\right)^{-1}. \tag{10}$$

Note that the first two coefficients of the β function, which are scheme-invariant, are $B(A)=(-3/2)A^2(1+A+\ldots)$.

Combining Eqs. (3), (4), and (9) for the $\hat{S}$ matrix, we find

$$B(A)\frac{d\hat{S}(A)}{dA}=\hat{\Gamma}\hat{S}-\hat{S}\hat{\gamma} \tag{11}$$

with the initial condition $\hat{S}(0)=1$.

Using (8), we can write the solution of Eq. (11) as

$$\hat{S}(A)=\begin{pmatrix} e & 0 \\ \frac{2B_0A}{B(A)}(1-e)-2e & \frac{B_0A^2}{B(A)Z(A)} \end{pmatrix}, \tag{12}$$

where $B_0=-3/2$ is the first coefficient of the β function, and

$$e\equiv\exp\left(-\int_0^A\frac{\gamma_{11}(Z^{-1}A)}{B(A)}dA\right).$$

We have taken into account the relationship between the β functions of the two schemes:

$$\frac{\beta(a)}{a}=\frac{B(A)}{A}\left(1-\frac{Z'}{Z}A\right). \tag{13}$$

To satisfy the initial condition, it is important that the expansion of γ_{11} start with $(-3/2)a^2$.

Substituting (12) into (10), we see that Eq. (10) holds identically for any γ_{11}, B, and Z in all orders of perturbation theory.

4. We can thus determine the explicit form of the transformation for the operators from the Adler-Bardeen scheme to the supersymmetry scheme. The axial anomalies (1) and (2) in this case are in agreement with each other, while the anomalous dimensions of the two currents differ from each other. There are no constraints imposed on the β function. The arbitrary constant $Z(A)$ can be determined from Eq. (13), for example. The axial anomaly can thus be solved by using transformations (9) and (12).

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