Field model with a dynamic cancellation of the cosmological constant

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A solution of the problem of the cosmological constant Λ is proposed. This solution is based on the introduction into the theory of a massless vector field, with a nonpositively determined energy density, which interacts with only the gravitation. The scalar condensate formed due to the divergence of this field cancels out Λ and gives rise to a power-law expansion regime.

The problem of the cosmological constant (see Ref. 1, for example) is currently one of the central topics of interest in cosmology and elementary particle physics. This problem can be summarized as follows. We know from astronomical observations that the vacuum energy density is negligible[2]:

$$\rho_{\text{vac}} < 10^{-47}\, m_N^4 \approx 10^{-47}\ \text{GeV}^4 \tag{1}$$

(ρ_{vac} is related to the cosmological constant Λ by $\rho_{\text{vac}} = m^2_{Pl}\Lambda/8\pi$, where $m_{Pl} \approx 10^{19}$ GeV). On the other hand, reasonable estimates of ρ_{vac} yield values (50–100) orders of magnitude larger than constraint (1). It is difficult to imagine that a random cancellation can have such a degree of accuracy, although attempts[1)] to find a logical explanation for the small value of ρ_{vac} can hardly be called successful yet.

A hypothesis that the cosmological constant vanishes dynamically due to the formation of a field condensate which cancels out the vacuum energy through the coupling to the space-time curvature (the back-coupling effect) seems to be attractive.[4] Regardless of whether such a hypothesis can be realized, all cosmological models based on it must have the following common feature: The cancellation of vacuum energy must be complete but only within a certain value on the order of the critical energy density $\rho_c = 3H^2 m_{Pl}^2/8\pi$.[3] Such a cancellation changes the established relationship between the age of the universe, t_U, and Hubble's constant H, may change the limit on the number of types of neutrinos,[5] and can explain to some extent the missing mass of the universe. A model of this mechanism, which was analyzed by Dolgov,[4] unfortunately cannot be formulated systematically on the basis of a scalar field. In this letter we show, however, that the vector field, which can be described by a simple Lagrangian, automatically cancels out a uniformly distributed energy density, ρ_{vac}, without canceling the gravitational interaction between material objects.

We assume that V_μ is described by the Lagrangian

$$\mathcal{L}(V) = \sqrt{g}\,[\eta\,(\frac{1}{2}\,(D_\alpha V^\alpha)^2 + \frac{1}{4}\,F_{\alpha\beta}F^{\alpha\beta}) + \xi R U\,(V^2)\,]\,, \tag{2}$$

where D_α is a covariant derivative, $F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$, R is the space-time curvature, η is a numerical constant, and $U(V^2)$ is a scalar function. We take the factor η to be $+1$ or -1, depending on the sign of ρ_{vac}. Accordingly, the sign of ξ should be chosen in a similar manner. We will not impose below the usual restriction $D_\alpha V^\alpha = 0$ on the states, but the lifting of this restriction does not lead to the appearance of negative transition probabilities, since the field V_μ appears quadratically in the Lagrangian. The energy density corresponding to this Lagrangian is not positive definite. However, since V_μ interacts only with the gravitation, this situation does not cause serious problems, manifesting itself only in that V_t increases with time in an asymptotically linear manner. To be consistent with observation, we must assume that $U(V^2)$ increases rather slowly as $V\rightarrow\infty$. We assume that

$$U(V^2) = \frac{m_1^2}{16\pi} \ln(1 + \frac{V^2}{m_0^2}). \tag{3}$$

We see from the standard scalar models in a flat and curved space that a summation of single-loop diagrams gives rise to a logarithmic field dependence of the effective potential. There is hope, therefore, that effective potential (3) can be found by starting with an interaction such as RV^2. This program, however, has not yet been realized. We will assume, nonetheless, that $U(V^2)$ is described by (3) and we will show that this model is unstable with respect to formation of the scalar condensate $(D_\alpha V^\alpha)$ which cancels the vacuum energy, so that $\rho_{vac} - \frac{1}{2}(D_\alpha V^\alpha)^2 \sim t^{-2}$. As a result, the expansion becomes a power-law expansion with Hubble's constant $H \sim t^{-1}$. We will examine this situation, for simplicity, by assuming a spatially flat universe and $\rho_{vac} > 0$, although the example in which $\rho_{vac} < 0$ seems to be more realistic. The latter case and other possible versions of the model will be examined in a more detailed paper.

The equations of motion for the field V_μ, which are derived from Lagrangian (2) with $\eta = -1$, are

$$\partial_\mu (D_\alpha V^\alpha) + D_\alpha F^\alpha_\mu + \xi R \frac{m_1^2}{8\pi} \frac{V_\mu}{V^2 + m_0^2} = 0 , \tag{4}$$

and the energy-momentum tensor corresponding to this Lagrangian is

$$T^{(V)}_{\mu\nu} = -\frac{1}{2} g_{\mu\nu} (D_\alpha V^\alpha)^2 + \frac{1}{4} g_{\mu\nu} F^2 - F_{\mu\alpha} F^\alpha_\nu$$
$$+ \partial_\beta (D_\alpha V^\alpha)(V_\mu \delta^\beta_\nu + V_\nu \delta^\beta_\mu - g_{\mu\nu} V^\beta) + \xi \frac{m_1^2}{8\pi}[R \frac{V_\mu V_\nu}{V^2 + m_0^2}$$
$$+ (g_{\mu\nu} D^2 - D_\mu D_\nu) \ln(1 + \frac{V^2}{m_0^2}) + G_{\mu\nu} \ln(1 + \frac{V^2}{m_0^2})] , \tag{5}$$

where $G_{\mu\nu} = R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R$; the last term therefore accounts for the time dependence of the gravitational constant, since $V^2 \sim t^2$, as will be shown below.

If $\xi R (V^2 + m_0^2)^{-1} < 0$, the number of solutions of Eq. (4) increases with time. Using Einstein's equation

$$\frac{m_1^2}{8\pi} G_{\mu\nu} = T^{(V)}_{\mu\nu} + g_{\mu\nu} \rho_{vac} + T^{(m)}_{\mu\nu} , \tag{6}$$

where $T^{(m)}_{\mu\nu}$ is the energy-momentum tensor for matter, we find the asymptotic expression

$$V_t \sim t + 0\left(\frac{1}{\xi^2 t}\right), \tag{7}$$

whereas the spatial part of $V_j V^j$ does not increase. Here $(D_\alpha V^\alpha)^2 \to 2\rho_{\text{vac}}$. This model, however, is not realistic, since the value of ξ for this sign of ξ $(\xi > 0)$ must be quite small: $\xi < 10^{-2}$ [otherwise $G(t)$ will change its sign]. On the other hand, a simultaneous solution of Eqs. (4)–(6) shows that $H \sim (\xi t)^{-1}$, so that although the expansion becomes a power-law expansion, its rate will be appreciably higher than that of the standard Friedmann cosmology.

If the field V_μ destroys the space-time curvature in cosmology by virtue of the scalar condensate $D_\alpha V^\alpha$, will it not also destroy the normal gravitational interaction observed in nature? Fortunately, the answer turns out to be negative, since in the inhomogeneous case the spatial derivatives in Eq. (4), which satisfies the field V_μ

$$D^2 V_\mu - R^\alpha_\mu V_\alpha + \xi R \frac{m_1^2}{8\pi} \frac{V_\mu}{V^2 + m_0^2} = 0, \tag{4'}$$

prevent the onset of instability. Only very strong gravitational fields might be an exception.

I am deeply grateful to Ya. B. Zel'dovich, a personal contact with whom was highly stimulating in many aspects of this study.

[1)]The literature on this subject is too voluminous to be included in this short letter; part of the literature may be found in a lecture delivered by Dolgov.[3]

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