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No-thinning simulations of extensive air showers and small scale fluctuations at the ground level

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The particle density in extensive air showers fluctuates at the ground level. These fluctuations, at the scale of the scintillator detector size (several meters), lead to the diversity of the individual detector responses. Therefore, small scale fluctuations contribute to the error in the estimation of the primary energy by a ground array. This contribution is shown to be non-Gaussian. The impact on the primary energy spectrum measured by a ground array is estimated. It is argued that super-GZK events observed by AGASA experiment do not result from the energy overestimation, due to small scale fluctuations, of lower energy events.

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1. Introduction. A significant part of conclusions on ultra-high energy cosmic rays (UHECRs) is made today on the basis of the quantities observed by ground detector arrays. Typical ground detector registers only small fraction ($< 10^{-6}$) of the shower particles at the ground level. The reading of the individual detector in the array is determined by the local density of particles in the shower. The latter is affected by the small scale fluctuations within the shower.

A typical extensive air showers (EAS), induced by a vertical proton with a primary energy of about 10^{20} eV contains about 20 billions of particles with energies above 1 MeV at the ground level. The modelling of such a huge number of particles is time and resource consuming process. Monte-Carlo simulations of air showers induced by ultra-high energy cosmic rays often involve procedures like "thinning" [1], aimed to reduce the effective number of particles in a calculation. These procedures make it impossible to estimate small scale fluctuations in a reliable way. Suggested "unthinning" procedure [2] washes out small scale fluctuations by its definition.

In the present work we use some artificial vertical proton-induced air-showers with energies up to 10^{18} eV, simulated without thinning, to estimate the impact of the small scale fluctuations on the energy spectrum in the ultrahigh-energy region, observed by a ground detector array with detectors simular to ones used in AGASA experiment.

The fluctuations were earlier estimated experimantally by the Akeno experiment [3]. The standard deviation value for the fluctuations was obtaned but the fluctuation distribution has not been discussed.

2. Energy estimation procedure. Ground array experiments traditionally use average density of scintillator signal $S(r_{\rm const})$, measured in units of vertical equivalent muons (VEMs), at a fixed constant core distance $r_{\rm const}$ as the energy estimator. The $S(r_{\rm const})$ is usually obtained by fitting the detectors readings by the empirical lateral distribution function (LDF) $S(r)$.

For example, AGASA experiment uses $r_{\rm const} = 600$ m and the following empirical LDF [4],

$$S(r) \propto \left(\frac{r}{R_M}\right)^{-1.2} \times \\ \times \left(1+\frac{r}{R_M}\right)^{-(\eta-1.2)} \left(1+\left(\frac{r}{R_1}\right)^2\right)^{-0.6}, \qquad (1)$$

where $\eta = 3.97 - 1.79(\sec\theta - 1)$, $R_M = 91.6$ m, $R_1 = 1000$ m. Only detectors with 300 m$\leq r \leq 1000$ m are used for the fit. Average scintillator detector responses for AGASA experiment are published in [5]. To ensure fit quality, AGASA implements the following procedure: the worst detector is excluded from the fit in the case of bad χ^2 ($\chi^2/N > 1.5$) [6] and the procedure is repeated until $\chi^2/N \leq 1.5$. Detectors experiencing largest local fluctuations are expected to be excluded by this procedure.

3. Simulations. We have generated several showers without thinning with primary energies ranging from 10^{17} to 10^{18} eV. The simulations were preformed with

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CORSIKA [7]. QGSJECT 01c [8], QGSJET II [9], GHEISHA [10] and EGS4 [11] models were used in simulations. The simulations without thinning are time consuming and CPU-time and storage required grows nearly linearly with primary energy. This is why we limit our simulations up to energies of 10^{18} eV.

All datafiles are made publicly available within the public library of artificial air showers called "Livni", so all the results of this and the following works may be confirmed using the same dataset. The library may be used for any other studies of the structure of air showers. The detailed information on the library content and access rules are available at the website *http://livni.inr.ac.ru*. Hereafter we use references to the library showers in the form of livni:codename, e.g. reference to the shower, named in the library as "18-3" will be livni:18-3.

4. Results. In the present work we intend to study fluctuations in air showers induced by the highest energy primary particles. Unfortunately, simulations of artificial air showers of such a high energy is impractical. In order to make statements on 10^{20} eV showers we consider scintillator detectors with an area 100 times larger than normal detector area (2.2 m^2 in AGASA) and utilize simulated showers with a primary energy of 10^{18} eV. This of course make our analysis approximate. The procedure is justified by the fact that the lateral distribution functions of scintilation signal density for energies 10^{18} and 10^{20} eV have a similar shape [4]. Furthermore, we have calculated the cross-correlation function,

$$C_S(\mathbf{d}) = \frac{\int d\mathbf{r} S(\mathbf{r})\ S(\mathbf{r}+\mathbf{d})}{\int d\mathbf{r} S^2(\mathbf{r})},$$

of scintillation density at the ground level. We found that cross-correlation function is close to zero on the scale of the detector size, $|\mathbf{d}| \gtrsim 0.4$ m.

The scintillation density distribution over detectors centered at the core distance between 595 and 605 meters is shown in Fig.1. To produce this plot we assume the ground to be completely covered by the detectors. It can be seen that an individual detector may be exposed to a larger or smaller density than an average one and the central part of the distribution obeys the Gaussian law in $\log(S)$ scale. Let us note that the plot refers to 600 m core distance where the fluctuations are small. At larger distances an individual detector may be exposed to up to 100 times larger particle density than an average, though the probability is small.

In order to reconstruct the observables we assume the ground array detector to consist of 100 plastic 5 cm thick scintillators (14.8 m × 14.8 m) forming square lattice covering the area of 100 km^2. Our intent is to make our detection procedure close to one used for the analysis of AGASA experimental data [12, 13]. The detector responses are estimated using AGASA average detector response functions [5]. Each simulated shower has been detected 30000 times with different core positions within the ground array and azimuthal angles with respect to the array.

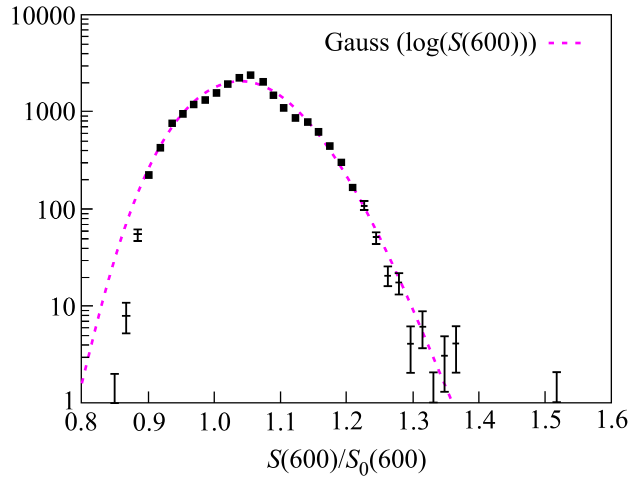

Fig.1. Scintillation density distribution over detectors (size of 14.8 m × 14.8 m) centered at [595 m; 605 m] core distance. Horizontal axis: scintillator density, normalized to average, vertical axis: number of detectors with a signal in a bin centered in $S(600)$ (livni:18-3 shower)

Fitting responses of the detectors at core distances from 300 to 1000 meters with the AGASA experimental LDF Eq. (1), we obtain $S(600)$. Following the AGASA procedure to ensure fit quality we exclude the worst detector from the fit in the case of bad χ^2 ($\chi^2/N > 1.5$) [6]. The distribution of the number of excluded detectors is presented in Table [2]).

The distribution of the number of excluded detectors in the fit quality assurance procedure (livni:18-3 shower)

Number of excluded detectors	% of cases
1	23%
2	15%
3	9%
4	7%
≥ 5	7%

The resulting distribution of reconstructed $S(600)$ calculcated for one artificial air shower is shown on Fig.2. The reconstruction error may depend on the first inter-

[2]) It should be noted that the distribution presented here is not neccesarily the same as in the analysis of the original AGASA data, as our definition of χ^2 does not include detector fluctuations and therefore is different from the experimental one.

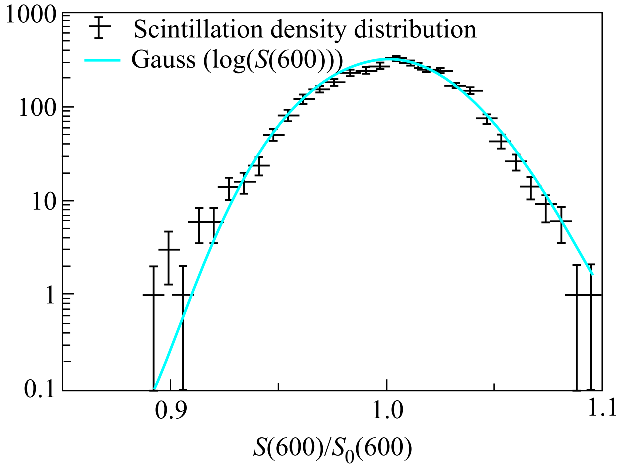

Fig.2. Distribution of reconstructed $S(600)$ normalized to the average value (livni:18-3 shower). Estimated energy distribution has the same form as an estimated energy is nearly proportional to $S(600)$

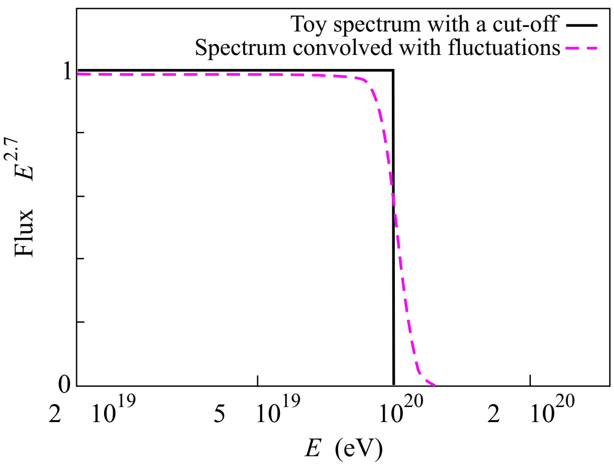

Fig.3. The impact of the small scale fluctuations on the toy primary spectrum with a cut-off. Dashed line shows the convolution of a toy spectrum and an energy estimation error presented in Fig.2

action producing a shower, as we discuss later in Sec. 5. The main part of the distribution may be fit with the Gaussian in $\log(S)$ scale. The same type of profile is suggested by AGASA for $S(600)$ experimental error distribution [13]. Finally, we estimate one-sigma error for $S(600)$ reconstruction for 10^{20} eV air showers due to small scale fluctuations as 7%. In rare cases $S(600)$ may be overesimated by factor of 1.5, hovewer, the probabilty of this is less than 10^{-4}. We may also see that the part of the distribution which corresponds to energy overestimation is broader than the underestimation part. The estimate does not include the fluctuations of the detector response, which are present in the experiment and are of the same order of magnitude [3].

The fluctuations discussed above may affect the primary spectrum observed by a ground array. Significant energy overestimation even in a relatively small number of cases may influence the experimental conclusion on the presence of the GZK cut-off [14, 15]. Let us assume a toy primary spectrum with a spectral density proportional to $E^{-\alpha}$, $\alpha = 2.7$ up to the energy of 10^{20} eV and equal to zero for higher energies. We have calculated the convolution of our toy spectrum and energy estimation fluctuation distribution (which is the same as $S(600)$ fluctuation distribution, presented in Fig.2). The resulting spectrum is shown in Fig.3. We see that the fluctuation contribution to the spectrum may be considered minor for the GZK-predictions: the probability of energy overestimation by a factor of 1.5 is less than 10^{-4}. We conclude that super-GZK events observed by AGASA experiment do not result from energy overestimation, due to small scale fluctuations, of lower energy events.

5. Limitations. One limitation of the above analysis is that is does not include the fluctuations of the detector response which are of the same order of magnitude [3]. The fluctuations within the detector may be accounted for by combining the library showers with relevant Monte-Carlo simulations of the detector.

The second problem one should care about is that the magnitude of small-scale fluctuations may be different for showers having different first interactions. To estimate this effect we analysed 20 showers with primary energy of 10^{17} eV and 3 showers with primary energy of 10^{18} eV. For each shower we have estimated $\alpha(r) = \sigma^2(r)/\bar{S}(r)$, where σ is a standard deviation of the detector response, measured in VEMs, at distance r (calculated for the ensemble of detectors centered at core distances close to r) and $\bar{S}(r)$ is the average detector response. As long as the correlation function C_S is zero $\alpha(r)$ doesn't depend on the detector size. This motivates the choice of $\alpha(r)$ as an estimator for the fluctuation magnitude in a particular artificial shower. For pure Poisson distribution (which would be the case for equivalent independent particles) $\alpha \equiv 1$. Average and maximum values of $\alpha(600)$ for 10^{17} eV showers are 0.47 and 0.51, respectively, with a standard deviation of 7%. Three studied 10^{18} showers have $\alpha(600)$ equal to 0.48, 0.57 and 0.69. These numbers imply that the difference between statistical properties of different showers is not very substantial. To interpret these data further, we assume that the character of the fluctuations does not change dramatically when the primary energy changes from 10^{18} to 10^{20} eV. As the study is based on a small number of artificial showers, we also have to assume practical inexistence of air showers with extremely large fluctuations. The first assumption may be checked by a simulation of 10^{20} eV artificial air shower without thinning and the second by simulating hundreds of showers without thinning. Both simulations are extremely resource consuming and yet are expected to be possible in the nearest future.

Thus, the difference in the fluctuation magnitude in different studied artificial showers is not substancial for the conclusions of the work, although our study is limited by the following:

- The primary energy of artificial showers does not exceed 10^{18} eV.
- The number of artificial events studied is relatively small and does not exclude the existence of paricular air showers with extremely large fluctuations in them.

The above limitations may be checked in the future.

6. Conclusions. We calculated the cross-correlation function of the scintillation signal density and found it to be close to zero at the detector scale $\gtrsim 0.4$ m. We estimated a contibution of small scale fluctuations on the detector scale to the energy reconstruction error by a ground array at the level of about 7% for primary energy of about 10^{20} eV. The contribution, although found to be non-Gaussian, is minor for GZK predictions. The study, however, has certain limitations; we discussed ways to get rid of them.

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