Supersolidity of glasses

A. F. Andreev[1]

Kapitza Institute for Physical Problems RAS, 119334 Moscow, Russia

Submitted 3 May 2007

Supersolidity of glasses is explained as a property of an unusual state of condensed matter. This state is essentially different from both normal and superfluid solid states. The mechanism of the phenomenon is the transfer of mass by tunneling two level systems.

1. Introduction. It was shown theoretically [1 – 3] that owing to the large probability of quantum tunneling of atoms, solid helium may be superfluid. All attempts to observe the superflow experimentally were unsuccessful (see [4] and [5]).

Kim and Chan [6] observed the reduction of solid ^{4}He rotational inertia below 0.2 K in the torsional oscillator experiments and interpreted it as the superfluidity of the solid. Further experiments (see [7] and references therein) show that the superfluid fraction observed for highly disordered (glassy) samples is remarkably large, exceeding 20%. This fraction seems to be absent in ideal helium crystals.

In 1972 it was shown [8, 9] that the quantum tunneling of the atoms explains some low temperature properties (thermal, electromagnetic, and acoustic) of glasses. The key point is the presence of the so-called tunneling two level systems (TLS) in the solid. A TLS can be understood as an atom, or a group of atoms, which can tunnel between two localized states characterized by a small energy difference.

In this paper we show that owing to the presence of coherent TLS's, quantum glasses manifest peculiar properties which are essentially different from those of normal and superfluid solids. Precisely these peculiar properties are observed experimentally (both [4, 5] and [6, 7]). At present the terms "supersolid" and "supersolidity" are used simply as synonyms for "superfluid solid" and "superfluidity of solids" respectively (see [10] for a review). We propose to use the term "supersolidity" to refer to the above mentioned properties of quantum glasses.

A normal solid is characterized by a single velocity of macroscopic motion: the solid bulk velocity **v**. The momentum density is $\rho\mathbf{v}$, where ρ is the mass density. The general motion of a superfluid solid is characterized (see [1]) by two mutually independent velocities: that of the solid bulk and the superfluid one. The supersolid (in our sense of the word) is characterized by a single velocity **v** of the solid bulk, but under certain conditions (see below) the momentum density is $(\rho - \rho_s)\mathbf{v}$, where ρ_s/ρ is the supersolid fraction. This is exactly what we need to explain both the reduction of rotational inertia [6, 7] and the absence of a superflow [4, 5]. We calculate ρ_s in terms of TLS parameters. The supersolid fraction, being proportional to the squared TLS tunneling amplitude, can be considerable for highly disordered solid ^{4}He and other quantum solids (hydrogen).

Our results are supported in recent experiment by Grigorev et al. [11]. They measured the temperature dependence of pressure in solid ^{4}He grown by the capillary blocking technique. At temperatures below 0.3 K (where the supersolidity was observed) they found the glassy $\propto T^2$ contribution to pressure. This is exactly what one expects from the TLS. On the other hand, the measurements of the melting pressure in perfect ^{4}He samples showed no deviations from $\propto T^4$ law [12].

2. TLS in moving glasses. The Hamiltonian H_0 of a given TLS in the frame of reference in which the solid bulk velocity **v** is zero, can be written as

$$H_0 = -\varepsilon\sigma_3 + J\sigma_1.$$

Here $\mp\varepsilon$ ($\varepsilon > 0$) are energies of two localized states, J is the tunneling amplitude, and σ_i ($i = 1, 2, 3$) are the Pauli matrices.

Let us suppose that the tunneling of the TLS is accompanied by displacement of a mass m by a vector **a**. The coordinates $\mathbf{r}_{1,2}$ of the center of gravity of the TLS before and after the tunneling can be written as $\mathbf{r}_{1,2} = \mp\mathbf{a}/2$. The operator form of the last equality is $\mathbf{r} = -\sigma_3\mathbf{a}/2$. The operator of velocity is determined by the commutator:

$$\dot{\mathbf{r}} = \frac{i}{\hbar}[H_0, \mathbf{r}] = -\frac{J\mathbf{a}}{\hbar}\sigma_2.$$

[1] e-mail: andreev@kapitza.ras.ru

The TLS momentum in the frame in which $\mathbf{v} = 0$, is

$$\mathbf{p} = m\dot{\mathbf{r}} = -\frac{mJ\mathbf{a}}{\hbar}\sigma_2.$$

In an arbitrary frame of reference a description of the TLS by means of a discrete coordinate is impossible. But we can use Galilean transformations to find the TLS Hamiltonian and momentum in the frame in which $\mathbf{v}$ is finite. We obtain

$$H_0 \to H_0 + \mathbf{pv} + mv^2/2,$$
$$\mathbf{p} \to \mathbf{p} + m\mathbf{v},$$

respectively. The last terms of both expressions must be included to the total kinetic energy and momentum of the solid bulk. Therefore, the contributions of the TLS tunneling to the energy and momentum of entire system are

$$H = H_0 + \mathbf{pv} \tag{1}$$

and $\mathbf{p}$, respectively. These two operators represent the energy and momentum of the tunneling TLS in the solid moving with velocity $\mathbf{v}$. Note that the operators $\mathbf{p}$ and H do not commute with each other.

The eigenvalues of the Hamiltonian H are $E_{1,2} = \mp E$, where $E = \left(\varepsilon^2 + \Delta^2\right)^{1/2}$, $\Delta = J\left(1 + u^2\right)^{1/2}$, and $u = (m/\hbar)(\mathbf{av})$. According to the general result of quantum mechanics ([13], §11) the mean values of momentum $\langle \mathbf{p} \rangle_{1,2}$ in the stationary states 1 and 2 are

$$\langle \mathbf{p} \rangle_{1,2} = \left\langle \frac{\partial H}{\partial \mathbf{v}} \right\rangle_{1,2} = \frac{\partial E_{1,2}}{\partial \mathbf{v}}.$$

We have

$$\langle \mathbf{p} \rangle_{1,2} = \mp \frac{J^2 m^2}{\hbar^2 E}\mathbf{a}(\mathbf{av}).$$

In the case of nonzero $\mathbf{v}$, the TLS has nonzero mean values of momenta in both stationary states. Note that in the TLS ground state, the projection of the momentum $\langle \mathbf{p} \rangle_1$ on the direction of velocity $\mathbf{v}$ is negative. This is the mechanism of supersolidity. The Hamiltonian H is identical to that of spin 1/2 in magnetic field. The sign of $\langle \mathbf{p} \rangle_1$ corresponds to Pauli paramagnetism.

3. Supersolidity. The equilibrium density matrix of the TLS (which is an almost closed system) in a uniformly rotating frame is

$$e^{(f' - H')/T},$$

where f' and H' are the free energy and Hamiltonian in this frame. The latter is determined by the expression

$$H' = H - \omega\mathbf{M} = H_0 + \mathbf{pv} - \omega\mathbf{M},$$

where ω is the angular velocity, $\mathbf{M} = \mathbf{R} \times \mathbf{p}$ is the TLS angular momentum, $\mathbf{v} = \omega \times \mathbf{R}$, and $\mathbf{R}$ is the TLS coordinate with respect to the rotation axis. We obtain $H' = H_0$. This means that a uniformly rotating supersolid behaves like a normal solid.

However, suppose that the solid bulk velocity depends on time $\mathbf{v} = \mathbf{v}(t)$ and is "switched on" adiabatically. This means (see [14], §11) that the switching time is much longer than the relaxation time in the solid but much shorter than the time during which the solid can be regarded as thermally insulated. The second of these two characteristic times is very long due to the Kapitza thermal resistance.

According to the general result of statistical mechanics ([14], §11 and §15) we have

$$\langle \mathbf{p} \rangle = \left\langle \frac{\partial H}{\partial \mathbf{v}} \right\rangle = \left(\frac{\partial f}{\partial \mathbf{v}} \right)_T,$$

where

$$f = -T\log\left(\mathrm{Tr}\,e^{-H/T}\right) \tag{2}$$

is the TLS free energy and H is determined by (1) with $\mathbf{v} = \mathbf{v}(t)$.

The free energy (2) can be written as

$$f = -T\log\left(e^{-E_1/T} + e^{-E_2/T}\right).$$

Here $E_{1,2} = \mp E$ are the eigenvalues of the Hamiltonian H. The mean value of the TLS momentum is

$$\langle \mathbf{p} \rangle = \frac{m\mathbf{a}}{\hbar}\left(\frac{\partial f}{\partial u} \right)_T.$$

Simple calculation gives

$$\left(\frac{\partial f}{\partial u} \right)_T = -\frac{J^2 u}{E}\tanh\frac{E}{T}$$

or

$$\langle p_i \rangle = -m_{ik}^{(s)} v_k,$$

where

$$m_{ik}^{(s)} = \left(\frac{Jm}{\hbar} \right)^2 a_i a_k \frac{\tanh(E/T)}{E}.$$

Let $Nd\varepsilon$ ($N = \mathrm{const}$) is the number of TLS's per unit volume of the solid and per interval of the energy half-difference $d\varepsilon$ near some ε which is much smaller than the characteristic height U of the energy barriers in the solid. The total momentum density $\mathbf{j}$ is

$$j_i = \rho v_i - \rho_{ik}^{(s)} v_k,$$

where the supersolid density tensor is

$$\rho_{ik}^{(s)} = \langle m^2 J^2 a_i a_k \rangle \frac{N}{\hbar^2} \int\limits_{\max(\Delta, T)}^{U} \frac{d\varepsilon}{\varepsilon}.$$

Here $\langle \ldots \rangle$ means the averaging over the ensemble of TLS's, and $\max(\Delta, T)$ is of the order of Δ if $T \ll \Delta$ and of the order of T if $T \gg \Delta$. Both T and Δ are much smaller than U.

For an isotropic system (glass) we have $\rho_{ik}^{(s)} = \rho_{\mathrm{s}} \delta_{ik}$, where

$$\rho_{\mathrm{s}} = \frac{N}{3\hbar^2} \langle m^2 J^2 a^2 \rangle \log \frac{U}{\max(\Delta, T)}.$$

We see that the characteristic temperature of supersolidity is of the order of Δ. The critical velocity v_c is determined by the condition $u_c \sim 1$. We have $v_c \sim \hbar/ma$. The critical velocities observed experimentally [6] are very small. This suggests the macroscopic character of the most effective TLS's. In principle, this is possible. The pressure dependence of ρ_s is determined by the competition of all parameters N, m, J, and a. Efficient TLS tunneling is facilitated by the presence of a region with reduced local density in the vicinity of the TLS. The ^{3}He impurity, due to the smaller mass of ^{3}He atoms, must bind to such regions (see [10]) destroying TLS. This is a simple explanation of supersolidity suppression by ^{3}He impurities observed in the experiments [6].

I thank L.A. Melnikovsky for helpful discussions. This work was supported by the Russian Foundation for

1. A. F. Andreev and I. M. Lifshits, Sov. Phys. JETP **29**, 1107 (1969).

2. G. V. Chester, Phys. Rev. A **2**, 256 (1970).

3. A. Leggett, Phys. Rev. Lett. **25**, 1543 (1970).

4. M. W. Meisel, Physica B **178**, 121 (1992).

5. J. Day, T. Herman, and J. Beamish, Phys. Rev. Lett. **95**, 035301 (2005); J. Day and J. Beamish, Phys. Rev. Lett. **96**, 105304 (2006).

6. E. Kim and M. H. W. Chan, Nature **427**, 225 (2004); Science **305**, 1941 (2004).

7. A. S. C. Rittner and J.D. Reppy, Phys. Rev. Lett. **98**, 175302 (2007).

8. P. W. Anderson, B. I. Halperin, and C. M. Varma, Phil. Mag. **25**, 1 (1972).

9. W. A. Philips, J. Low Temp. Phys. **7**, 351 (1972).

10. N. Prokofev, arXiv: cond-mat/0612499v1.

11. V. N. Grigorev, V. A. Maidanov, V. Yu. Rubanskii et al., arXiv: cond-mat/0702133.

12. I. A. Todoshchenko, H. Alles, H. J. Junes et al., cond-mat/0703743, Pis'ma ZhETF **85**, 555 (2007).

13. L. D. Landau and E. M. Lifshits, *Course of Theoretical physics*, Vol. 3, *Quantum Mechanics: non-relativistic theory*, Pergamon Press, Oxford, New York, 1977.

14. L. D. Landau and E. M. Lifshits, *Course of Theoretical physics*, Vol. 5, *Statistical Physics*, Butterworth, London, 1999.