Diffraction focusing of neutrons

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We show the possibility of diffraction focusing of neutron Bloch waves inside a crystal in a manner similar to diffraction focusing of a free electromagnetic field by means of Fresnel zone plates and lenses. Analytical and numerical calculations predict the possibility of increasing flux density of collimated monochromatic neutrons by up to five–six orders of magnitude.

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It has been shown[1-3] that in the case of hard x rays the Bloch wave flux inside a crystal may be focused with order-of-micron accuracy if the wave packet incident on the focusing crystal is appropriately modulated. In particular, a second crystal with the same thickness and orientation as the first may be used as a possible modulator. Radiation density enhancement in the focal region was approximately two orders of magnitude higher than the densities of the background and the wave packet incident on the focusing crystal. One practical application of this effect is a spectrometer with diffraction focusing of spectral lines.[4] Similar experiments with thermal neutrons were recently conducted in a number of laboratories abroad,[3] and the results were presented at the Symposium on Neutron Interferometry (Grenoble, June 1978).[5]

We should emphasize that the diffraction focusing was heretofore obtained only in the case of modulation of an incident wave by one-or two-crystal plates (normally all plates are cut from a single crystal and form a common piece of II- or III-shaped structure, where the first plates modulate the wave packet and the last one focuses it). In particular, all efforts to produce an analog of a plate or Fresnel lens for the hard x-rays have been unsuccessful.[2]

We should note, however, that the transition from x rays to thermal neutrons gives rise to a qualitatively new experimental situation. Inasmuch as absorption of the neutron wave field in a crystal falls off in comparison with the x-ray case by more than three orders of magnitude while the dynamic scattering parameters remain qualitatively the same, it is possible to use plates several centimeters thick which correspond to broad Fresnel zones for the wave field of Block waves in a crystal, and to use thin layers of a resonant absorber (e.g., cadmium, gadolinium) for modulating the amplitude of the primary packet. If the normal zone plates for the electromagnetic field are designed in accordance with the known weighting function $G(\mathbf{r}) = r^{-1}\exp(i\mathbf{kr})$—which describes a freely-propagating spherical wave—the weighting function in each scattering plane for the dynamically Laue-diffracted hard radiation is[6]

$$G(z,\,x) = \frac{i\pi}{2\,\Lambda\,\mathrm{tg}\,\theta}\; J_0\left(\frac{\pi}{\Lambda}\sqrt{z^2 - x^2\,\mathrm{ctg}^2\theta}\right),$$

where θ is the Bragg angle, z is distance from the crystal surface, x is distance along the diffraction vector, Λ is the extinction length which in a typical case (reflection Si(220), thermal neutrons with $\lambda = 1.2$ Å) is 122 μm.[7] The Fresnel zones lie between the Bessel function zeros $J_0(\pi/\Lambda)(z^2 - x^2\text{ctg}^2\theta)^{1/2}$. The entire scheme is shown in Fig. 1. The first analysis of the effect as applied to the x-ray case was carried out at the author's request by Chukhovskiĭ who confirmed the existence of the effect but failed to assess the sharpness of focusing (see review[8]). In the case of neutrons, for which the plate thickness $t \gg \Lambda$, we may use the known asymptotic behavior of the Bessel function and, using the normal method of evaluating integrals of rapidly-oscillating functions, we get the following expression for the central peak amplitude in a diffracted beam $E_1(x)$, normalized to unit amplitude of a plane wave incident on a crystal:

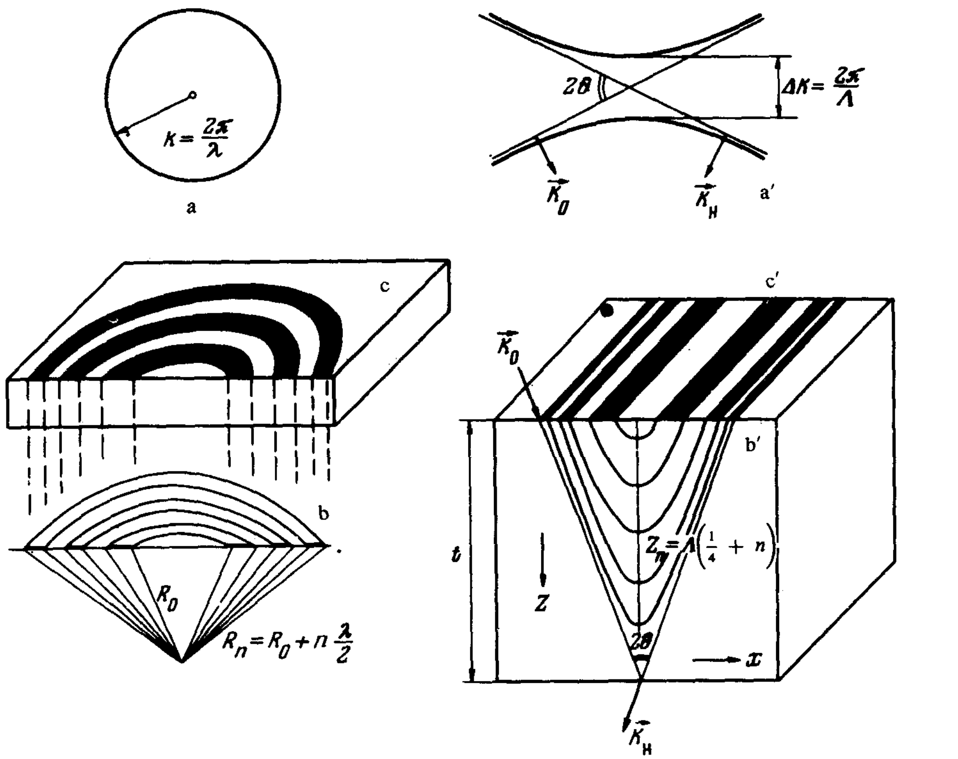

FIG. 1. Scheme of Fresnel zones and zone plates for particles (left) and quasi-particles (right): a—dispersion surface $\omega(\mathbf{k}) = \text{const}$ for free particles in a vacuum; a′—the same for quasi-particles (i.e., Bloch waves) near the Brillouin zone boundary, i.e., in a region of dynamic scattering for thermal neutrons: $\Delta\mathbf{K} \simeq 10^{-6}$ $\mathbf{k}$, $\Lambda \simeq 10^6 \lambda$; b—construction of Fresnel zones for case a. Spherical wave in a vacuum, weighting function $G(\mathbf{r}) \sim r^{-1}\exp(i\mathbf{kr})$; b′—the same for case a′. A plane wave with wave vector $\mathbf{k}_0$ is incident on a crystal with thickness t with reflecting surfaces perpendicular to crystal surface $z = 0$. For a diffracted wave with wave vector $\mathbf{k}_H$, the weighting function $G(z,x) \sim I_0(\pi/\Lambda)(z^2 - x^2\cot^2\theta)^{1/2}$; c—normal Fresnel zone plate for the case b;c′—zone plate for the Bloch waves in the case b′.

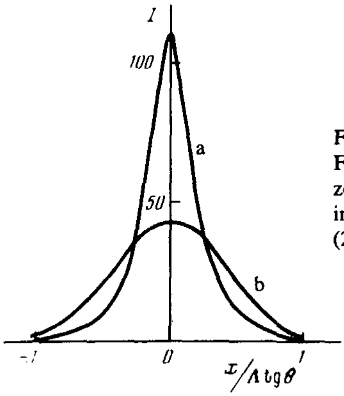

FIG. 2. Focal spot intensity $I=|E_1(x)|^2$ for a zone plate of the type in Fig. 1c′: a—using total number of zones ($\xi=1$); b—using central zones ($\xi=1/(2)^{1/2}$). Intensity was normalized to unit amplitude of incident plane wave, thermal neutrons with $\lambda=1.2$ Å, reflection Si (220), $\Lambda=122$ μm, plate thickness $t=400\,\Lambda$.

$$|E_1(0)| = \int_{-t}^{t} iG(t,x)\Theta[iG(t,x)]dx \simeq \frac{1}{\pi}\sqrt{\frac{2t}{\Lambda}}\int_0^1 \frac{du}{\sqrt[4]{1-u^2}}$$
$$= \frac{2}{\pi}\sqrt{\frac{t}{\Lambda}}\left\{2E\left(\frac{\pi}{2},\frac{1}{\sqrt{2}}\right)-F\left(\frac{\pi}{2},\frac{1}{\sqrt{2}}\right)\right\} \cong 0.54\sqrt{\frac{t}{\Lambda}}, \quad (1)$$

where the Θ-function describes a zone plate ($\Theta(x)=1$ and 0 for the positive and negative arguments, respectively), and F and E are partial integrals of the first and second kind, respectively. If only the central zones of a plate are used, which occupy a region $|x/t|<\xi\tan\theta$, the expression contained inside the braces in Eq. (1) becomes

$$\left\{2E\left(\phi,\frac{1}{\sqrt{2}}\right)-F\left(\phi,\frac{1}{\sqrt{2}}\right)\right\}, \quad (2)$$

where $\phi=\arccos(1-\xi^2)^{1/4}$. In particular, using zones with a width that exceeds $\Lambda\tan\theta$ and neglecting narrower zones $\xi\simeq 1/(2)^{1/2}$, we get instead of Eq. (1)

$$|E_1(0)| \cong 0.33\sqrt{\frac{t}{\Lambda}}. \quad (3)$$

The accuracy of Eqs. (1–3) was confirmed by detailed computer calculations which also produced a flux intensity distribution of the diffracted wave in the focal region $I=|E_1(x)|^2$ (Fig. 2). In the case of crystals with 5-cm thickness, $(t/\Lambda)^{1/2}=20$ and, correspondingly, the growth of the central peak intensity $|E_1(0)|^2$ (i.e., the increase in the flux density of collimated monochromatic neutrons) attains almost two orders of magnitude even when narrow zones are neglected. Two focusing crystals, 4–5 cm thick and placed one after the other so as to cross-over the reflecting planes, amplify the flux density of monochromatic neutrons by 3–4 orders. The half width of the focal spot is approximately 0.4 $\Lambda\tan\theta$ in the case of Eq. (1) and $\Lambda\tan\theta$ in the case of Eq. (3).

The flux density of monochromatic neutrons may be further increased by means of switching from the Fresnel zone plate analog to the Fresnel diffraction lens analog for the Bloch wave field. In order to achieve this, it is necessary to shape the surface of the focusing crystal according to a particular law, corresponding to the function $G(z,x)$. Numerical calculations indicate a possibility of increasing the flux density in this case up to six orders of magnitude.

The foregoing results indicate the possibility of producing diffraction-neutron optics and carrying out experiments with tightly-collimated thermal neutrons, which were heretofore outside the realm of possibility.

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