Phase analysis of the $\pi^+\pi^0$ and $\pi^-\pi^0$ interactions in the reaction $\pi^\pm p \to \pi^\pm \pi^0 p$

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Results are presented on the phase analysis of $\pi^+\pi^0$ and $\pi^-\pi^0$ scattering obtained by means of energy-dependent and energy-independent methods.

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As was pointed out earlier in Ref. 1, the $\pi\pm\pi^0$ states are rarely used in the phase analysis of the $\pi\pi$ scattering. Meanwhile, when studying such states one may simultaneously obtain the δ_0^2, δ_1^1 and δ_2^2 phases and study the effect of the interference terms using the D_2 wave as an example. Today, when in order to advance the physical understanding of $\pi\pi$ interactions we must improve the accuracy and reliability of results, information on the $\pi^\pm\pi^0$ scattering may be extremely useful.

Our work is based on findings obtained in the course of exposing a liquid-hydrogen bubble chamber to π^- and π^+ meson beams with momenta of 4.5 GeV/c and 3.05 GeV/c, respectively. Both the reactions $\pi^- p \to \pi^-\pi^0 p$ (4960 events) and $\pi^+ p \to \pi^+\pi^0 p$ (2600 events) were processed using the same methodology.

One of the important questions to be addressed in a study of $\pi N \to \pi\pi N$ reactions is the question concerning the mechanisms of non-single-pion exchange, in particular the formation of isobars in the final state. In the case of the reaction $\pi^- p \to \pi^-\pi^0 p$ at $p_{\pi^-} = 4.5$ GeV/c the Dalitz curves fail to show a noticeable isobar contribution, while for the reaction $\pi^+ p \to \pi^+\pi^0 p$ at $p_{\pi^+} = 3.05$ GeV/c formation of the Δ^{++} (1232) and Δ^+ (1232) isobars is discernible. Therefore, events occurring in a region $1185 < m_{\pi p} < 1285$ were selected out and were not used in the subsequent analysis.

We used the cross-sections $\sigma_{\pi^-\pi^0}$ and $\sigma_{\pi^+\pi^0}$ and the averaged spherical harmonics $\langle Y_L^0 \rangle (t, m_{\pi\pi})$, $L = 0,\ldots.4$ as experimental data for the phase shift analysis. All these quantities must be known at the mass shell, i.e., at the pion pole. The $\sigma_{\pi^-\pi^0}$ cross section was obtained through extrapolation using a pseudo-peripheral approximation in the region $|t| < 0.3$ (GeV/c)^2.[1] In order to improve the statistics of the interaction cross-section $\sigma_{\pi^+\pi^0}$, we used a wider region $|t| < 0.5$ (GeV/c)^2.

As is known, contributions of non-single-pion exchange mechanisms in the $|t| > 0.3$ (GeV/c)^2 region are substantial. In order to eliminate these components we used the Van Hove angle ω for the sampling.[2] Angle ω is defined as

$$\omega = \mathrm{arctg} - \frac{\sqrt{3}\, q_p}{q_{\pi^+} - q_{\pi^0}} \qquad \text{(See Fig. 1)}$$

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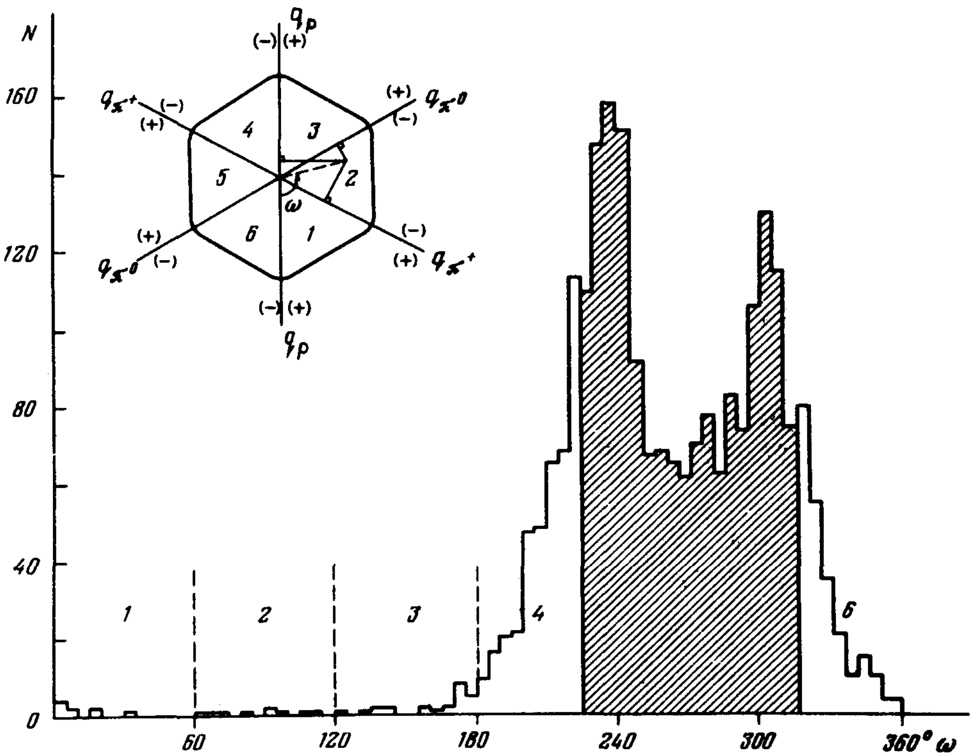

FIG. 1.

where q_i is momentum of secondary particle i in the center system.

It was shown in Ref. 1 that single-pion exchange predominates in one of the six sectors in the Van Hove diagram which corresponds to a region $240° < \omega < 300°$ in our coordinates for the reaction $\pi^+ p \to \pi^+ \pi^0 p$. To carry out the work, we selected a broadened region $225° < \omega < 315°$ (cross-hatched histogram area in Fig. 1). One would hope that single pion exchange events are enriched in this region and that their extrapolation to the pion pole would yield correct results.

In the case of events due to the reaction $\pi^+ p \to \pi^+ \pi^0 p$ extrapolation was carried out by two methods: using a normal function $F'(t) = F(t)/t$, i.e., as a pseudo-peripheral approximation, and using the conformal transformation variable x.[4] The latter may be defined as

$$x = x(t) = \frac{at + b}{t + c},$$

where a, b and c parameters are picked so as to transform the physical region in the t-plane into a $(-1, 1)$ region in the x-plane. In our case the values of these parameters were: $a = 3.08$, $b = -c = 0.521$ $(\text{GeV}/c)^2$. In both cases we used a linear function with a sufficiently good value χ^2. Figure 2 shows the values of cross sections—● for $\sigma_{\pi^- \pi^0}$ and Δ for $\sigma_{\pi^+ \pi^0}$—obtained by means of $F(t)/t$ and $F(x)/t$, respectively. Good

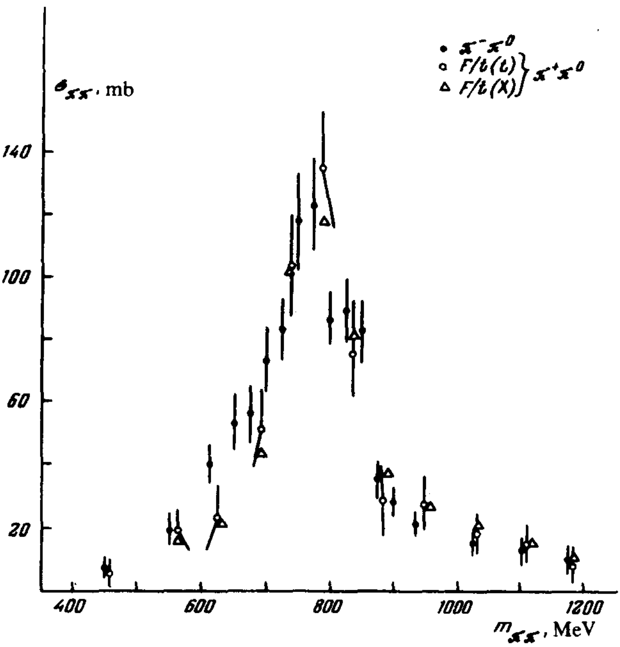

FIG. 2.

agreement of all the results is evident. The extrapolated values of averaged spherical harmonics for both reactions are shown in Fig. 3.

The phase analysis was carried out for the $360 < m_{\pi\pi} < 1300$ MeV region using both energy-dependent and energy-independent methods. In the energy-dependent analysis all experimental points are approximated concurrently. The minimizing functional is as follows

$$W = \sum_{i=1}^{N} \frac{(\sigma_i^{\text{theor}} - \sigma_i^{\text{exp}})^2}{\Delta\sigma_i^{\text{exp}}} + \sum_{l=1}^{4} \sum_{i=1}^{N} \frac{(< Y_l^0 >_i^{\text{theor}} - Y_l^0 >_i^{\text{exp}})^2}{\Delta < Y_l^0 >_i^{\text{exp}}},$$

where N is the number of mass intervals. The interaction was considered to be elastic, and cross sections and harmonics were set as functions of δ_0^2, δ_1^1 and δ_2^2 phases. The exact formulas, and a description of the energy-independent method, may be found in Ref. 1. The phases were parametrized in the following manner:

$$\delta_0^2 = \begin{cases} a_0 + a_1 m_{\pi\pi} + a_2 m_{\pi\pi}^2 \\ \operatorname{arcctg}\left\{\frac{1}{k a_0} + \frac{1}{2} k a_1\right\} \end{cases} \text{or}$$

$$\delta_1^1 = \mathrm{arctg}\,\frac{a_3 a_4}{a_3^2 - m_{\pi\pi}^2}$$

$$\delta_2^2 = a_5 + a_6\, m_{\pi\pi},$$

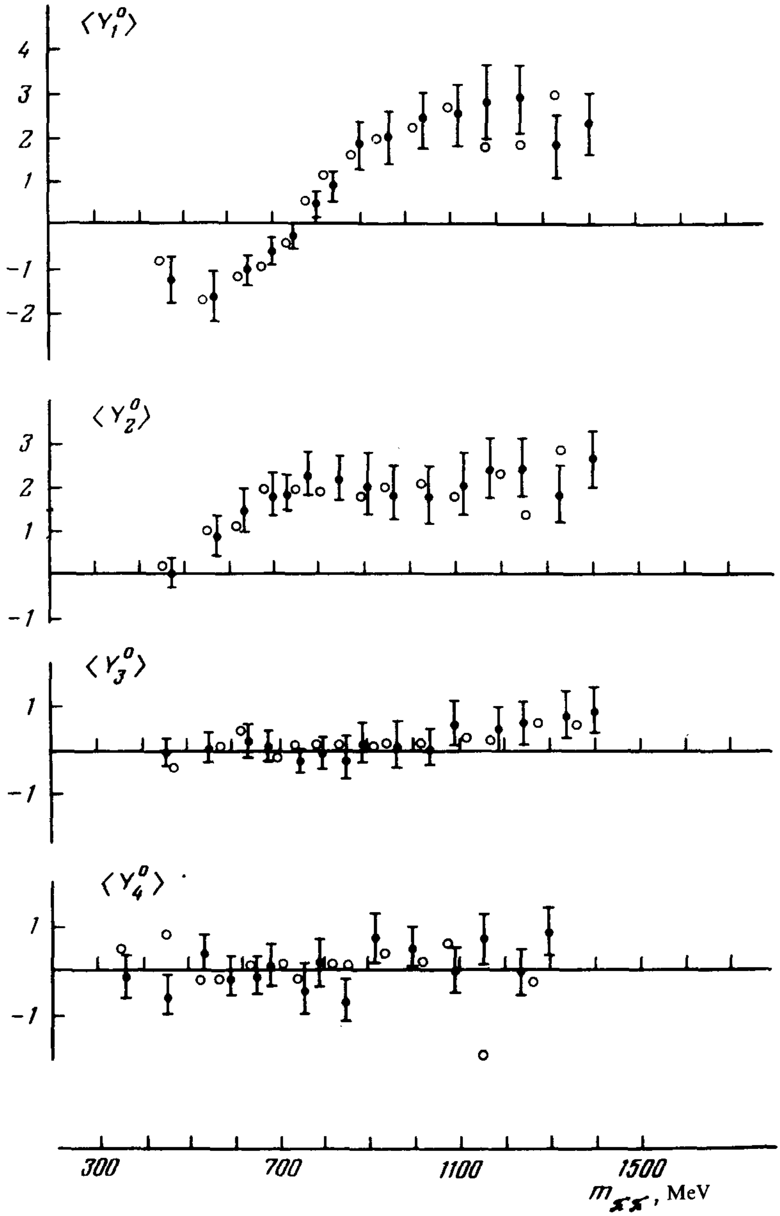

FIG. 3.

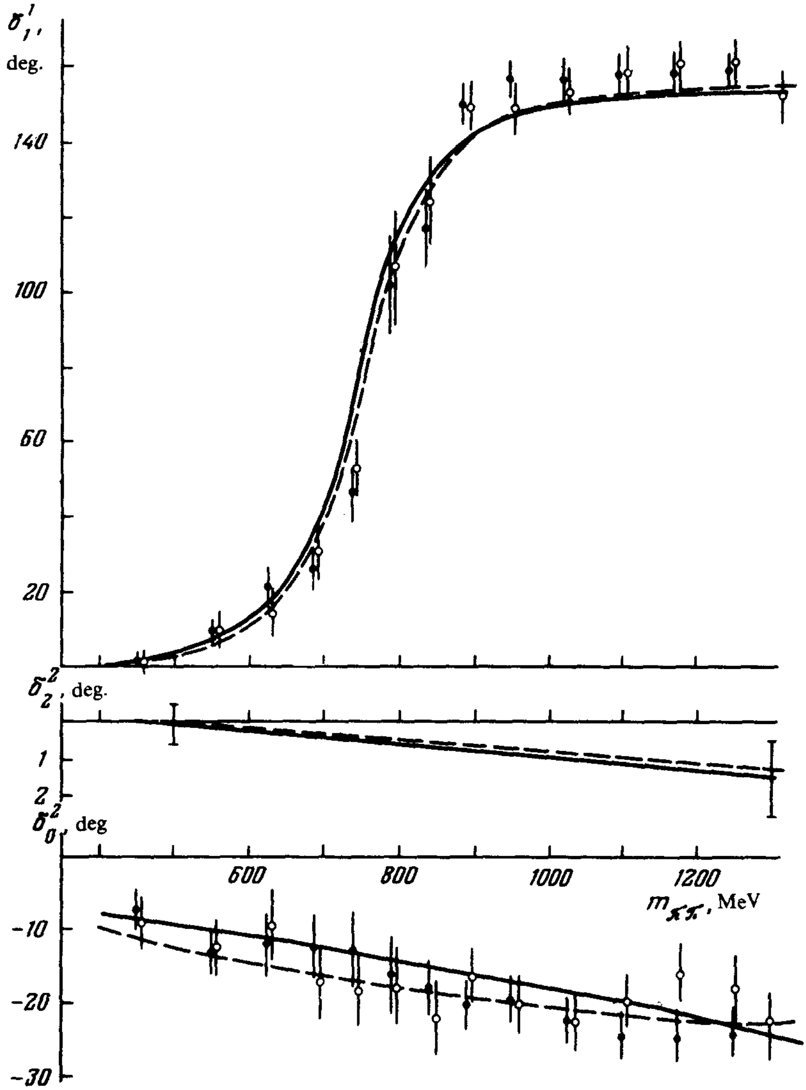

FIG. 4.

where a_i are free parameters, $k = [(m_{\pi\pi}^2/4) - \mu_\pi^2]^{1/2}$, and μ_π is ion mass.

Figure 4 shows the resultant phase curves. Clearly, within the limits of experimental error agreement exists between results obtained from the energy-dependent and energy-independent analyses, and also between the data from various reactions. Changing the form of phase parametrization does not lead to considerable deviation from the foregoing values. The $\rho^{\pm}$-resonance parameters are stable:

$$m_{\rho^-} = (762 \pm 4)\ \text{MeV};\quad \Gamma_{\rho^-} = 142 \pm 8\ \text{MeV};\quad m_{\rho^+} = 772 \pm 4\ \text{MeV};$$

$$\Gamma_{\rho^+} = 141 \pm 8\ \text{MeV}$$

The energy-independent analysis yields the following results in terms of an effective radius approximation for the S_2-wave scattering length:

$$a_0^2 = (-0.131 \pm 0.014)\mu^{-1}; \qquad r_0^2 = (-0.112 \pm 0.046)f$$

which is in agreement with Ref. 5.

The phase curves for S_2 and D_2 waves are in fair agreement with the global data for $\pi^{\pm}\pi^{\pm}$ scattering, although δ_2^2-phase values normally decrease sharply, attaining $\sim -3°$ at 1200 MeV.[6]

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