Quark plasma instantons, super-heavy hadrons, and neutron stars

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(Submitted 24 November 1978)

Pis'ma Zh. Eksp. Teor. Fiz. **29**, No. 1, 115–118 (5 January 1979)

Analysis of the contribution of instantons to the equation of state of a quark plasma leads to a conclusion concerning the existence of the plasma – hadron gas phase transition of the first kind. Comparison of calculations with data on neutron star masses excludes the existence of multi-baryon hadrons composed of light quarks.

PACS numbers: 11.10.St, 12.40.Bb, 21.65. + f, 97.60.Jd

In our preceding work[1] we showed that the nonlinear fluctuations of the calibration field—instantons—may be suppressed in a medium with relatively high baryon charge density n_B, temperature T or in a strong chromatic field $F^a_{\mu\nu}$.[2] An important consequence of this phenomenon is displacement of quarks and gluons from the vacuum into "bags"[3] which represent strongly-interacting hadronic particles.

Earlier comparison of characteristics of nuclear matter[4] in the quark plasma—calculated within the framework of the theory of perturbations[5]—showed that transition between these states of the matter occurs at a density $n_B \sim n_0 = 1\Phi^{-3}$. In this work, we allow for the instantons which permits us to approach the transition region $n_B \sim n_0$ more closely and to give more reliable reasons for the nature of this transition. In particular, we offer arguments in favor of this transition being the phase transition of the first kind with a finite density drop. The production of metastable hadrons with large baryonic charge is possible for certain parameters, although calculations of the critical masses of neutron stars and the analysis of existing data[6] tend to exclude this possibility.

Let us begin with qualitative considerations concerning the role of instantons in nuclear matter. Inasmuch as instanton radii in a vacuum as $\sim 1\Phi$,[7] instantons are somewhat suppressed in a gap between the nucleons up to center-to-center distances of 2–3Φ. The vacuum pressure gradient between the outer and inner sides of the nucleaons leads to an effective attraction. We currently don't know what portion of the single-nucleon potential well $U \approx 40$ MeV is associated with this effect at a nuclear density $n_N = 1017\Phi^{-3}$. However, it is clear to us that this effect sets in rapidly with increasing density and at $n_B \gtrsim n_0$—when considerable fractionof instantons is suppressed—the effective potential $U \sim \epsilon_0/n_0 \sim 400$ MeV, where ϵ_0 is energy in the vacuum.[7,11] Thus, damping of the equation of state is predicted at $n_B \sim n_0$.

Let us now proceed to a quantitative analysis of the equation of state in the quark phase., The energy density ϵ and pressure p may be expressed as follows:[11]

$$\epsilon = \epsilon^{(TP)} + B - B C n_B^{-5/3} \qquad (1)$$

$$p = p^{(TP)} - B + \frac{8}{3} B C n_B^{-5/3},$$

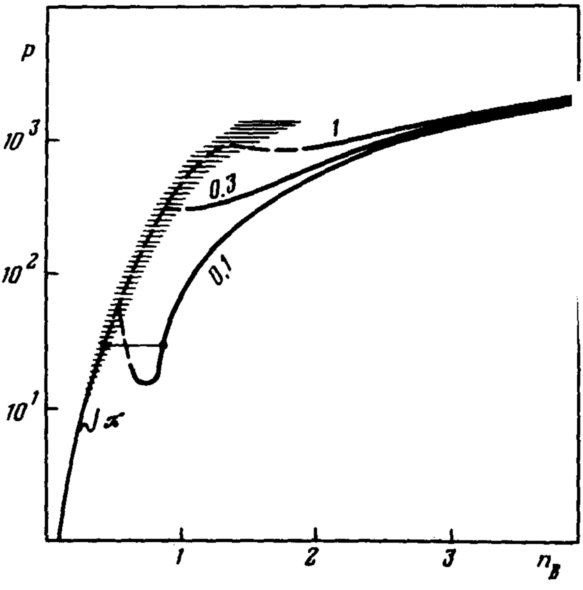

FIG. 1. Dependence of pressure p (MeV/Φ^{-3}) on baryon charge density n_B (Φ^{-3}). Continuous curves correspond to Eq. (1), numbers at the curves—values of parameter C (Φ^{-3}). Cross-hatched region corresponds to calculations of nuclear matter[4], dashed curves—assumed shape of curve. Points on lower part of curve designate the boundary of region of stability. π-designated curve corresponds to allowance for π-condensation.

where $\epsilon^{(TP)}$ and $p^{(TP)}$ are the first terms of a series in the theory of perturbations calculated in Ref. 5: $(a_s \equiv g^2(p_F)/4\pi)$

$$p^{(TP)} = \frac{3\pi^{2/3} n_B^{4/3}}{4}\left[1 - \frac{2a_s}{\pi} + \frac{a_s^2}{\pi^2}\ln a_s + \dots\right] \approx \frac{1}{3}\epsilon^{(TP)} . \qquad (2)$$

In Ref. 2 we neglected the quark masses u, d, s. The second terms[1] correspond to energy and pressure in a vacuum:[7,1]

$$B = |\epsilon_0| \approx 450\ \text{MeV}/\Phi^{-3} = 0.0035\ \text{GeV}^4 . \qquad (3)$$

We should note that the phenomenological value of B [3] is several-fold smaller. Finally, the last term defines contributionof the plasma instantons in accordance with Ref. 1. At large values of n_B this contribution is small, and is reflected by the suppression of instantons. For the present, the parameter C can not be theoretically determined (see Ref. 1) and remains free, its value being of the order of $\sim 1\Phi^{-5}$.

Figure 1 shows the function $p(n_B)$ for various values of C. A minimum occurs at $n_B \sim n_0$ as a result of existence of bound states of this density. At $C < 0.095$, the minimum attains a zero value which suggests the possibility of existence of hadrons with

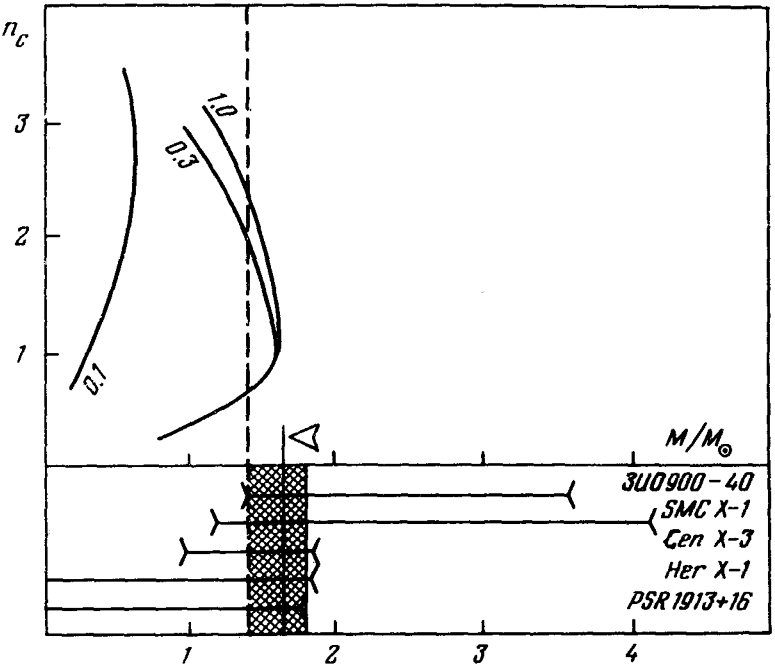

FIG. 2. Dependence of the central density of a star n_c (Φ^{-3}) on its mass M (in solar mass units $M_\odot$). Lower part of figure shows experimental limitations[6] on masses of binary pulsars. Dashed vertical lines denote greatest lower boundary; arrow and solid vertical line denote maximum mass obtained from calculations.

large baryonic charges. Such hadrons are metastable in any case, since their decay into nucleons is energetically advantageous.

The applicability of Eq. (1) ends at densities at which the second and third terms become equivalent. In this case, the number of instantons in a plasma is almost the same as in a vacuum, and interactions among term must be taken into account. At this time we do not know how this may be accomplished, but in the physical sense it is evident that there occurs a transition to an "anomalous" phase: nucleon gas. Figure 1 shows the hypothetical shape of the curves (dotted lines). The break represents a phase transition of the first kind which is associated with color "capture".

It is significant that matter in this region is unstable and remains unrealizable, since the conventional phase transition of the first kind takes place with a density jump (see Fig. 1). The pressure corresponding to the transition point is determined by the conventional rule of Maxwell's planes. For this reason the pressure, and especially the density jump are relatively insensitive to curve details.

Thus, given the foregoing stipulations we may consider the presence of a phase transition of the first kind as believable at $C \lesssim 1$; this may serve as an additional argument in favor of a model of a sufficiently hard sack for light-quark hadrons.

The critical mass of cold stars represents a quantity that is very sensitive to the equation of state in the density range of interest to us. Figure 2 shows the solution of the Tallman-Oppenheimer-Volkov equation. The various curves correspond to different values of parameter C, and a region beyond the maximum is unstable with respect to collapse into a black hole. The lower part of Fig. 2 shows the experimental limitations on the mass of binary pulsars.[6] The cross-hatched region is the most probable of all the available data. The vertical dashed line at $M = 1.4\ M_{\odot}$ correspondsto the lower mass boundary of the heaviest star. Variants which fail to reach this boundary contradict the experiment. Consequently, we may conclude that $C \gtrsim 0.15$ which leads us to the following significant physical corollary: super-heavy hadrons composed of light quarks do not exist. Experiments being readied on the collision of high-energy heavy ions are expected to verify this conclusion.

Upon completion of this work we became aware of the work of Callan *et al.*[8] which basically postulates the following: suppression of instantons inside hadrons as a cause for the formationof "bags" is indentical to that presented in our earlier work.[1] Callan and coworkers[8] also studied effects due to instanton interaction and identified a phase transition of the first kind from the external field which qualitatively is similar to transition described above. In addition to this, they theoretically evaluated the parameter $B = 130\bar{\mu}^4$ which, at the recommended value of $\bar{\mu} = 73$ MeV yields $B = 0.0027$ GeV^4 which is in a good agreement with values in Ref. 3—obtained by entirely different methods[7,11]—and is in a sharp contrast with the phenomenological value 0.0005 GeV.[3] This agreement is important not only from the viewpoint of the present work, since B represents the fundamental constant of the physics of strong interactions.

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[1] E.V. Shuryak, Suppression of instantons as an origin of quark confinement, Novosibirsk preprint 78-54, May 1978; Phys. Lett., in press.

[2] A.M. Polyakov, Phys. Lett. **59B**, 82 (1975); A.A. Belavin, A.M. Polyakov, A.A. Schwartz, and Yu. S. Tyupkin, ibid. **59B**, 85 (1975).

[3] A. Chodos, R.L. Jaffe, K. Johnson, C.B. Thorn, and V.F. Weisskopf, Phys. Rev. **D9**, 3471 (1974).

[4] V.A. Pandaripande, Nucl. Phys. **A178**, 123 (1971); H.A. Bethe and M.B. Johnson, ibid. **A230**, 1 (1974); I.D. Walecka, Ann. Phys. N.Y. **83**, 491 (1974).

[5] I.C. Collins and M.I. Perry, Phys. Rev. Lett. **34**, 1353 (1975); M. Nauenberg and G. Chapline, Nature **264**, 235 (1976); G. Baym and S.A. Chin, Phys. Lett. **62B**, 241 (1976); E.V. Shuryak, Zh. Eksp. Teor. Fiz. **74**, 408 (1978) [Sov. Phys. JETP **47**, 212 (1978)]; B.A. Fridman L.D. McLerran, Phys. REv. **D16**, 1130, 1147, 1169 (19700; V. Baluni, ibid. **D17**, 2092 (1978).

[6] P.C. Joss and S.A. Rappaport, Nature **264**, 219 (1976).

[7] A.I. Vainshtein, V.I. Zakharov and M.A. Shifman, Pis'ma Zh. Eksp. Teor. Fiz. **27**, 60 (1978); Preprints ITEP, 73, 80, 81, 94, (1978).

[8] G. Gallan, R. Dashen and D.J. Gross, IAS Preprint "Theory of hadronic structure," June 1978.