Concerning the process $\nu + N \to \nu + l^+ + l^- +$ (hadrons) at high energies

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We show that deeply-inelastic lepton pair production is suppressed in comparison with hadronless processes up to energies of $\dot{E} > 10^5$ GeV.

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The process of direct production of lepton pairs in the Coulomb field of the nucleus belongs to the number of calculable processes. From the beginning,[1] it was noted that the nuclear form factor plays an important role in this process, lowering the rate of growth of cross section with energy. Therefore, it appears interesting to know whether the rate of growth lost due to form factor is not regained in the course of transition to deeply-inelastic reactions.

Numerical calculations,[2] carried out for energies of the order of several GeV showed that the cross section with hadron production is substantially less than the cross section without hadron production. Is this a general rule for all energies or is it the result of the lack of energy for the full development of the hadron jet at an initial energy of the order of GeV's? This question is also important because in the case of rapid growth of the deeply-inelastic cross section the mechanism in question could be used as a source of a certain portion of so-called dilepton events. In this paper we report on having obtained an answer to this question in an analytical form and on the basis of an approximation method that has worked well earlier.[3,4] The approximation consists of replacing the square of the matrix element, which corresponds to the lepton vertex in the diagram, with a cross section for the photoproduction process

$$\nu + \gamma \to \nu + l^+ + l^-,$$

which in the case of a muon pair production is[3]1)

$$\sigma_f = \begin{cases} \dfrac{4\alpha G^2}{9\pi^2} w^2\left(\ln\dfrac{\sqrt{w^2}}{\mu} - \dfrac{65}{48}\right), & w >> \mu \\ \\ \dfrac{2^{15/2}\alpha G^2}{\pi^2 7!!} w^2\left(\dfrac{w-2\mu}{w}\right)^{7/2}, & \nu \gtrsim 2\mu \end{cases}, \tag{1}$$

where w is total 4-momentum of the final leptons. The cross section of a deeply-inelastic process involving a proton is defined by the following formula

$$d\sigma^{\rm inel} = \frac{\alpha}{\pi}\frac{d\Delta^2}{\Delta^2}\frac{\nu W_2(x,\Delta^2)}{x}dx\,\sigma_f$$
$$\times\left\{\frac{1}{w^2+\Delta^2} - \frac{w^2+\Delta^2}{4E^2\Delta^2} - \frac{1}{2mEx} + \frac{w^2+\Delta^2}{8m^2E^2x^2}\right\}dw^2, \tag{2}$$

where m is proton mass, $\Delta^2 = -q^2$, $x = \Delta^2/2mq_0$, E is lab. energy of initial neutrino, $\nu W_2(x,\Delta^2)$ is structure function of deeply-inelastic scattering.

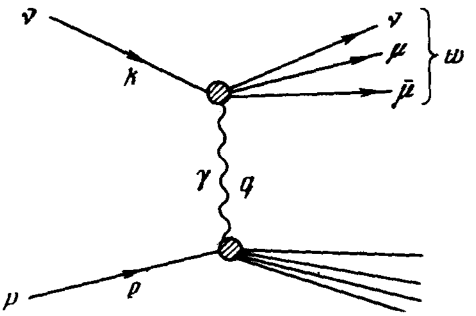

FIG. 1.

In the event that the interaction with a proton is elastic, a similar formula for the cross section—obtained as a result of neglecting the anomalous magnetic moment—follows:

$$d\sigma^{\rm el} = \frac{\alpha}{\pi}\frac{d\Delta^2}{\Delta^2}F^2(\Delta^2)\,\sigma_f\left\{\frac{1}{w^2+\Delta^2} - \frac{w^2+\Delta^2}{4E^2\Delta^2} - \frac{1}{2mE} + \frac{w^2+\Delta^2}{8m^2E^2x^2}\right\}dw^2, \tag{3}$$

where $F(\Delta^2)$ is the proton form factor.

In order to understand what gain in the energy growth of the cross section is accrued where the form factor vanishes, we shall examine the result of integration of the first (basic) term inside the braces in Eq. (3). For the sake of simplicity, we shall consider the form factor as follows:

$$F(\Delta) = \begin{cases} 1, & \Delta \lesssim \Delta_o \\ 0, & \Delta > \Delta_o \end{cases},$$

and, noting that in a broad range of values $\sigma_f \sim w^2$, we shall substitute $\tilde{\sigma}_f = cw^2$ for σ_f.[2)] Thus,

$$\tilde{\sigma} \approx \frac{ac\, mE}{\pi} \left\{ 4 \ln \frac{\Delta_o + \sqrt{\Delta_o^2 + 4m^2}}{2m} + \frac{\Delta_o}{m} \left(\sqrt{\frac{\Delta_o^2}{m^2} + 4} - \frac{\Delta_o}{m} \right) \right\}.$$

The above formula yields an important conclusion: the presence of the form factor leads only to a logarithmic decrease in the rate of growth of cross section since

$$\tilde{\sigma} \approx \begin{cases} \frac{2ac\, mE}{\pi} 2 \frac{\Delta_o}{m}, & \Delta_o << m \\ \frac{2a\, cmE}{\pi} \left(\ln \frac{2E}{m} - 1 \right), & \Delta_o = \Delta_{max} = \sqrt{2mE} \end{cases} \tag{4}$$

Thus, in the case of proton for which $\Delta_0 \approx m/2$, the gain derived from the vanishing of the form factor is reduced to only the occurrence of a multiplier $\log(E/m)$ in the total cross section. In order that the process of lepton pair production in the deeply-inelastic interaction may exceed the production process in the elastic interaction, it is necessary that the factor $\log(E/m)$ exceed the numerically small multiplier that results from integration of the cross section (Eq. (2)) with respect to x. The experimental data for $E = 270$ GeV shows,[5] that for large values of Δ^2, the function νW_2 rapidly decreases with increasing x. The following approximate expression may be used for evaluation:

$$\nu W_2(x) \approx \begin{cases} (0.6 - x), & x \lesssim 0.6 \\ 0, & x > 0.6 \end{cases}. \tag{5}$$

Upon integration we affirm that the deeply-inelastic production of lepton pairs may exceed "elastic" production under the condition

$$0{,}18 \ln \frac{E}{m} > 1. \tag{6}$$

The above condition fails, however, to reflect the fact that, in reality, in place of the photon cross section in Eqs. (2) and (3) we have function $a(w^2, \Delta^2)$ which at $\Delta^2 = 0$

is tranformed into the photon cross section, and at $\Delta^2 \neq 0$, it decreases with increasing Δ^2 approximately as $f = [w^2/(w^2 + \beta\Delta^2)]^{\gamma}$ as is shown in the numerical calculations in Ref. 6, where β and γ are numbers of the order of unity.[3)] Allowance for f was not essential when calculating the cross section of "elastic" production, since the form factor led to a cut-off $\Delta^2_{\rm eff} \ll w^2_{\rm eff}$. In the case of inelastic production $\Delta^2_{\rm eff} \sim w^2_{\rm eff} \sim Em$, and Eq. (6) must be substituted with the following condition

$$0.18 < f > \ln\frac{E}{m} > 1 . \tag{7}$$

Numerical calculations at $E = 20$ GeV show[6] that $\langle f \rangle \lesssim 0.5$. At $\langle f \rangle = 0.5$, we get $0.09 \log (E/m) > 1$, or $E > 6 \times 10^4$ GeV.

[1)]In Ref. 3, a factor of 2 is missing from these formulas.

[2)]This approximation holds true for determining the relative value of cross sections in Eqs. (2) and (3).

[3)]The conclusion concerning the decrease in $a(w^2, \Delta^2)$ with increasing Δ^2 results from analysis of the behavior of functions $V_1(\Delta^2)$ and $V_2(\Delta^2)$ in Ref. 6.

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