CP violation and the Cabibbo angle in the quaternion model

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The quark mass matrix in the gauge model of the quaternion fields is investigated. The model requires introduction of eight quark states. The discrete M symmetry, which does not allow mixing of "light" u, d, s, and c quarks with the "heavy" t, b, t', and b' quarks, is discussed. A spontaneous *CP* violation in the peaks of the interaction of the Higgs bosons with the scalar quark current $\bar{d}s$ occurs in the sector of the "light" quarks. The Cabibbo angle $\theta_c = \arctan(m_d/m_s)^{1/2} - \arctan(m_u/m_c)^{1/2}$ is calculated within the limit of *CP* invariance.

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The gauge theory of the quaternion fields (Q fields) of spins 0, 1/2, and 1 was recently examined by us as a unified theory of weak and electro-magnetic interactions of leptons and quarks.[1] Just as the complex field in the Lagrangian indicates the presence of an electric (or some other additive) charge in the theory, the quaternion field seems to correspond to a "weakly electric" charge (Q charge) whose components produce SU(2) ⊗ U(1) algebra. In the simplest case, the theory contains one left-handed quaternion-field of spin 1/2 (L), which may be linked to a quartet of left-handed quarks or leptons. Generally, the other quartets of quarks and leptons have an arbitrary number of such L fields. These quartets, including the quartet of scalar fields, break up into pairs of isodoublets if the mass parameters and interaction constants of the corresponding Q fields in the Higgs and Yukawa[2] sectors are quaternions.[1] On the other hand, the right-handed quaternion fields (R), instead of possessing a total Q charge (like the L fields and the scalar Q field), presumably have only an electric charge (which is the same for all components of the given R field[1]) and the quartets corresponding to them break up into isosinglets—the right-hand components of the quark and lepton fields. Thus, it is clear that a consistent classification of quarks is possible only if there are two[2] L fields, $L^{(1)}$ and $L^{(2)}$, which induce the same quartets of particles with a doublet-doublet filling

$$\bar{\bar{L}}^{(1)} = \left[\begin{pmatrix} u \\ d \end{pmatrix}\begin{pmatrix} c \\ s \end{pmatrix}\right]_L, \qquad \bar{\bar{L}}^{(2)} = \left[\begin{pmatrix} t \\ b \end{pmatrix},\begin{pmatrix} t' \\ b' \end{pmatrix}\right]_L \tag{1}$$

and two R fields, R_u and R_d, which yield two different quartets—a u type (with a charge $+2/3$) and a d type (with a charge $-1/3$), respectively, which have only weak isosinglets

$$\bar{\bar{R}}_u = \begin{pmatrix} u & c \\ t & t' \end{pmatrix}_R, \qquad \bar{\bar{R}}_d = \begin{pmatrix} b & b' \\ d & s \end{pmatrix}_R \tag{2}$$

where t' and b' are the seventh and eighth quarks, which distinguish the Q model from the generally accepted three-doublet model.[3] Everything said here about quarks also pertains to leptons with the exception that in the case of leptons one R field with a charge -1 is introduced in order to give mass to the e, μ, τ, and τ' leptons and take it away from the ν_e, ν_μ, ν_τ, and $\nu_{\tau'}$ neutrinos.

Let us examine the scalar field $\phi = \phi_0 + ie_k\phi_k$, $k = 1,2,3$ (henceforth the recurrent indices in the equations denote summation). The polynomial for the Higgs field ϕ has the standard form[3]:

$$P_\phi = \frac{1}{2} m^2 \phi^+ \phi + \frac{1}{2} h (\phi^+ \phi)^2 + Q.c. \tag{3}$$

Polynomial (3) for the four components $\{\phi_0, \phi_1, \phi_2, \text{and } \phi_3\}$ of the field ϕ, which are written as two isodoublets D_1 and D_2, has the form:

$$\frac{1}{2} P(D_1, D_2) = \frac{1}{2} m_0^2 (\bar{D}_1 D_1 + \bar{D}_2 D_2) + h_{03}^- (\bar{D}_1 D_1)^2 + h_{03}^+ (\bar{D}_2 D_2)^2$$
$$+ h_{12} (\bar{D}_1 D_1 + \bar{D}_2 D_2)(\bar{D}_1 D_2) + h_{12}^* (\bar{D}_1 D_1 + \bar{D}_2 D_2)(\bar{D}_2 D_1) + 2 h_0 (\bar{D}_1 D_2)(\bar{D}_2 D_1), \tag{4}$$

where $h_{03}^\pm = h_0 \pm h_3$, $h_{1,2} = -(h_1 - ih_2)$, and, for simplicity (and without loss of generality), we assume that $m_k = 0$ and $m = m_0$.[4] The fields D_1 and D_2 for $m_0^2 < 0$ develop vacuum expectations: $\langle D_1 \rangle = (\mathring{\lambda}_1)$, $\langle D_2 \rangle = (\mathring{\lambda}_2) e^{i\epsilon}$ and $D_m = \langle D_m \rangle + d_m$, $(m = 1,2)$. The mass matrix of the fields d_1 and d_2 can be calculated explicitly. Diagonalizing it by going over to the new fields $\tilde{d}_1$ and $\tilde{d}_2$,

$$\tilde{d}_1 = d_1 \cos a + d_2 e^{-i\delta} \sin a, \qquad \tilde{d}_2 = -d_1 \sin a + d_2 e^{-i\delta} \cos a \tag{5}$$

[it turns out that $\delta = \epsilon$ and $\tan \alpha = (\lambda_2/\lambda_1)$], after Higgs procedure,[2] we obtain five massive scalar bosons $\tilde{d}_2^{\pm}$, $\text{Im}\tilde{d}_2^0$, and $\text{Re}\tilde{\tilde{d}}_{1,2}^0$, where $\approx$ denotes rotation similar to that in Eq. (5) but with a different angle α' (although the phase is the same, $\delta = \epsilon$).

The original Su(2) ⊗ U(1) gauge symmetry (Q symmetry) is spontaneously broken in the subgroup of the electric charge.[1]

Let us examine the mass matrix of the quarks. The most common Q-invariant[1] coupling of the L and R fields with the scalar field ϕ has the form:

$$\mathcal{L}_y = [G^{pq}_{mn}, L^{(m)}\phi_q]_p R^{(n)}_q + \bar{R}^{(n)}_q [\phi^+_q L^{(m)}, G^{pq}_{mn}]_p + Q.c., \qquad (6)$$

where $m, n = 1, 2$; $q = u, d$; $p = \pm$ (commutator, anticommutator), $\phi_u = \phi^*$, $\phi_d = \phi$, $R^{(1)}_q = (1 - ie_3/2)R_q$, $R^{(2)}_q = (1 + ie_3/2)R_q$, and $(1 \pm ie_3/2)$ are projection operators which "extract" from the R_q fields the "light" u, c, d, and s (and the "heavy" t, t', b, and b') components. The nondiagonal elements of the quaternion coupling matrices $G^{pq}_{mn}(m \neq n)$ mix two quartets of quarks—"light" and "heavy." We introduce a discrete symmetry M, which prohibits this mixing:

$$L^{(1)} \to iL^{(1)}, \quad L^{(2)} \to -iL^{(2)}, \quad R_{u,d} \to e_3 R_{u,d} \qquad (7)$$

(the latter transformation in terms of the $R^{(1)}$ and $R^{(2)}$ fields indicates that $R^{(1)} \to iR^{(1)}$ and $R^{(2)} \to -iR^{(2)}$). The invariance of the total Lagrangian relative to the transformations[7] leads to a diagonal Yukawa coupling in the quark Q fields,[6] $G^{pq}_{12} = G^{pq}_{21} = 0$. As a result of a vacuum shift of the scalar fields, the mass matrix of the light quarks (after the usual procedure[1] of multiplying the Q fields in coupling (6) and using the components) can be written as follows:

$$\mathcal{L}_M = \overline{S^q_{mL}}\,(\lambda_1 A^q_{mn} + e^{i\epsilon_q}\lambda_2 B^q_{mn}) S^q_{nR} + h.c. \qquad (8)$$

$[S^u = (u/c), S^d = (d/s), \epsilon_u = -\epsilon, \epsilon_d = \epsilon]$, where A^q_{mn} and B^q_{mn} are components of the matrix elements G^{pq}_{mn}, which satisfy the conditions: $A^q_{12} = (A^q_{21})^*$, $B^q_{11} = B^q_{22}$, $B^q_{12} = B^q_{21} = 0$, and $A^q_{11} = 0$, if in the G^{pq}_{mn} quaternions the zero component conincides with the third component ("0 − 3" symmetry). Although diagonalization of Z_M in the quark fields cannot cause[3] CP violation in the usual weak interaction with intermediate vector bosons $W^\pm$ and Z^0, it induces it in the peaks of the interaction of the Higgs bosons Im$\tilde{d}^0_1$ and Re$\tilde{\tilde{d}}^0_{1,2}$ with the scalar quark current $\bar{d}s$. Because of a large number of parameters in $P(D_1, D_2)$ and Z_M, this violation can always be arranged so as to give the necessary value of $(K^0_1 - K^0_2)$ mixing. An analogous approach can be used for the heavy quarks t, b, t', and b'.

In the absence of M symmetry, CP violation occurs in sector (6) even for exact SU(2) ⊗ U(1). Moreover, when it is spontaneously broken, CP violation also occurs in the usual weak peaks with $W^\pm$ and Z^0 bosons because of mixing of four quarks of u and b type, respectively. This, however, seems to us less attractive. On the other hand, the M symmetry localizes the weak interaction in the quartet quarks that are isolated from each other (exact Cabibbo universality) and, in accordance with it, the experiments on neutrino production of heavy hadrons containing t, b, t', and b' quarks should give a negative result. At the same time, the (e^+e^-) annihilation does not preclude their production.

Let us return, however, to the mass matrix (8). If in the approximation the CP-noninvariant terms in Z_M are missing, $B^q_{11} = B^q_{22} = 0$, and the bare mass of the u and

d quarks is equal to zero, $A^{q}_{11} = 0$ ("0 − 3" symmetry), then the physical (diagonalized) quark fields have the following mass ratios

$$m_u = \tan^2\theta\, m_c \;, \quad m_d = \tan^2\theta' m_s \;, \tag{9}$$

which gives for the Cabibbo angle $\theta_c = \theta' - \theta$ the values cited in the footnote,[5)] in good agreement with experiment.[5]

Note that both results—*CP* violation and the Cabibbo angle—were not specially chosen[4,5] couplings in the Higgs and Yukawa sectors, but were determined from the Q structure of the fields and parameters of the theory as a result of a simple (standard) choice of these couplings.

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[1)]It should be remembered, in contrast to the L fields and the scalar field which are transformed with the Q phase, the R field undergoes only the usual phase tranformation, which is the same for all its components.[1]

[2)]Note that with one L field and one R field we would have four massless—two right-handed-quark states.

[3)]Here, $m^2 = m_0^2 + ie_k m_k^2$, $= h_0 + ie_k h_k$, $(m^2)^* = m^2$, and $h^+ = h$ (" + ") denotes total Hermitian (*) and quaternion (Q.c.) conjugation.[1]

[4)]This potential was postulated by Sikivie[4] for the purpose of violating the *CP* invariance in the Weinberg-Salam model.[2]

[5)]This value of θ_c was obtained within the frame of the $SU(2)_L \otimes SU(2)_R \otimes U(1)$ theory in a number of studies (see, for example, Ref. 5). In the $SU(2) \otimes U(1)$ theory, this result was obtained for the first time because the symmetry of the Yukawa potential is stronger in the Q model than in the standard Weinberg-Salam model[2] with the same particle content.

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