# Čerenkov amplification of fast waves in solids of revolution

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A wave of arbitrary large velocity, which propagates along the axis of a circular cylinder, can be radiated and amplified according to the Čerenkov mechanism by a current which flows along the surface of the cylinder perpendicularly to its axis.

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As is well known, Čerenkov instability occurs when the emitter (a particle, an electron cluster, drift flux, etc.) moves in the direction of propagation of the wave with a velocity exceeding the phase velocity of this wave. It is therefore assumed that on the basis of the Čerenkov principle only the slow waves or those that are purposely slowed down can be amplified in a solid in which the velocity of the emitter does not appreciably exceed $\sim 10^7 - 10^8$ cm/sec. We show here, however, that Čerenkov amplification of waves with arbitrary large velocities, which propagate along the symmetry axis, can occur in solids of revolution.

Let us examine, for example, an infinitely long, circular cylinder of radius $R$ and a wave that propagates along the $z$ axis of the cylinder. Because of the presence of symmetry, the dependence of the wave field on the polar angle $\phi$ and on time $t$ can have the form $\sim \exp i(n\phi - \omega t)$ with $n = 0, \pm 1, \pm 2, \ldots$, i.e., it is a wave that propagates along the azimuth. The Cerenkov principle when applied to this wave suggests that its instability can be induced by creating a circular current of electrons on the surface of the cylinder in a plane perpendicular to $z$. In this case only the angular velocity (rather than the linear velocity, as is usually the case) of the emitter (drift flux) must exceed the phase velocity $d\phi/dt = \omega/n$, i.e.,

$$\frac{v}{R} > \frac{\omega}{n}, \tag{1}$$

where $v$ is the drift velocity. Thus, the Čerenkov condition (1) includes the radius $R$ and the mode number $n$, but *does not include* the wave number $q_z$, which determines the true phase velocity of the wave in the direction of the $z$ axis, $v_\Phi = \omega/q_z$. Therefore, the waves with arbitrarily large $v_\Phi$ also satisfy condition (1).

  

To show that amplification can be achieved by overtaking the azimuthal wave by the electrons, we examined a specific case of an exchangeless spin wave propagating along the axis of a ferro-magnetic cylinder magnetized to saturation in the field $\mathbf{H}_0 \| z$.[1] The cylinder, which was coated with a semiconducting layer, has a slit to which a constant voltage was applied to produce a ring current. If the condition $q_z R \ll 1$ is satisfied, the media are isotropic in the plane perpendicular to the $z$ axis, the width of the slit is $\ll R$, only the components of the field $E_{0\phi}$ and of the current $j_{0\phi}$ are nonvanishing in the static state, and the Hall effect is weak-$(\mu_H H_0/c) \ll 1$ ($\mu_H$ is the Hall mobility and $c$ is the velocity of light), then the Maxwell equations and their boundary conditions in the cylindrical coordinates $(R,\phi,z)$ can be broken down into two sets of equations that determine independently the $TM$ and $TE$ waves. The spin waves can be only of $TM$ type, since magnetization in them precesses in the $\perp \mathbf{H}_0$ plane and the hf field components $\delta H_R$, $\delta H_\phi$, and $\delta E_z$ are nonvanishing. The instability condition can be written in the form $\alpha = \alpha_e + \alpha_m < 0$, where the electron absorption coefficient

$$\alpha_e = \frac{\frac{1}{2} \int dV \mathrm{Re}\, [\delta \mathbf{j}\, \delta \mathbf{E}^*]}{\frac{1}{16\pi} \int dV \left[ \frac{d(\omega\epsilon)}{d\omega} |\delta \mathbf{E}|^2 + \frac{d(\omega\mu_{xx})}{d\omega} |\delta \mathbf{H}|^2 \right]}, \tag{2}$$

$\epsilon(\omega)$ and $\mu_{xx}(\omega)$ are the dielectric constant and permeability, integration is carried out over the total volume, $\delta_j$ is the current density fluctuation, the magnetic loss $\alpha_m = |\gamma| \Delta H/\pi$, $\gamma$ is the gyromagnetic ratio, and $\Delta H$ is the width of the FMR line in the ferrite. It follows from the phenomenological expression for the current[2] that in our case the only nonvanishing component is

$$\delta j_z = \sigma\left(\delta E_z - \frac{v}{c}\delta H_R\right) = \sigma \delta E_z\left(1 - \frac{nv}{\omega R}\right), \tag{3}$$

where $\sigma$ is the conductivity and $v = -\mu_H E_{0\phi}$. We can see from Eq. (3) that the current $\delta_{jz}$ is comprised of the ohmic current produced by the $\delta E_z$ field and the current produced by the Lorentz force

$$\frac{1}{c}[\mathbf{v}, \delta \mathbf{H}]_z = -\frac{v}{c}\delta H_R,$$

whose phase at $v > 0$ is opposite to that of $\delta E_z$. It can be seen from Eq. (3) that if condition (1) is satisfied for the outer radius of the semiconductor layer, then the $\delta j_z$ phase will be opposite to that of $\delta E_z$ in the bulk of this layer and the electrons will generate the energy of the wave. This proves that Čerenkov radiation in this case is possible.

Let us now evaluate the absolute value of $\alpha_e$ (2). We note that $|\delta E| \sim (\omega R/nc)|\delta H| \ll |\delta H|$ and the frequency $\omega = \omega_0 = [\omega_H(\omega_H + \omega_m)]^{1/2}$ (Ref. 1), where $\omega_H = |\gamma| H_0$, $\omega_m = 4\pi|\gamma| M_0$, and $M_0$ is the saturation magnetization. The rearrangement of the spectrum of the spin wave is weak due to its interaction with the electrons, which is ensured by the condition $(R/l_{sl})^2 \ll 1$, where $l_{sl} = c/(4\pi\sigma\omega)^{1/2}$ is the depth of the skin layer. Thus, from Eq. (2) we obtain

$$\alpha_c \sim \frac{2\omega}{n}\left(\frac{R_{eff}}{l_{sl}}\right)^2\left(1-\frac{nv}{\omega R_{eff}}\right)\left[1+\left(\frac{V_F}{V_n}\right)\left(\frac{d(\omega\mu_{xx})}{d\omega}\right)_{\omega=\omega_0}\right]^{-1},$$

where $R_{eff}$ is the effective radius of the semiconducting layer and $V_\Phi$ and $V_n$ are the effective volumes occupied by the field in the ferrite and semiconductor. The maximum energy $|\alpha_e|$ is generated at $R_{eff} = nv/2\omega$. If $V_\Phi \sim V_n$ and $[d(\omega\mu_{xx})/d\omega] \approx 2(\omega_H \ll \omega_m)$, then $|\alpha_e|_{max} = nv^2/6\omega l_{sl}^2$. At 300 K for $n$-GaAs with the electron concentration $\sim 10^{16}$ cm$^{-3}$ and mobility $\sim 10^4$ cm$^2$/V-sec we have $v \sim 10^7$ cm/sec and $l_{sl} \sim 10^{-2}$ cm. For ZIG at a frequency $\omega_0 \sim 10^{10}$ sec$^{-1}$ we have $\Delta H < 0.5$ G. Then the condition for amplification, assumes the form $|\alpha_e|_{max} \sim 1.6 \times 10^7\, n$ sec$^{-1} > \alpha_m \sim 2.5 \times 10^6$ sec$^{-1}$, and can be satisfied, for example, at $n = 10$. In this case $(R_{eff}/l_{sl})^2 \approx 0.2 \ll 1$ and $R_{eff} \sim 50\,\mu$m. Thus, the described amplification of the fast spin wave apparently can be obtained experimentally. Note that if the cross section of the cylinder is not an ideal circle, then the azimuthal waves, which propagate in the opposite direction, will be produced. For a square cross section we obtain a pure standing wave $\sim \cos(n \cos\phi)e^{-i\omega t}$. Since the waves, which propagate against the drift, are dampened, their presence makes it more difficult to achieve instability.

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