Singularities of the angular distributions of elastically scattered isobaric nuclei of the neighboring elements (a possible method of studying low-energy pion-nucleus interaction)

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We predict a new effect—the oscillator structure in the angular distribution of elastically scattered isobaric nuclei of the neighboring elements. The search for this effect is important because of the possibility of developing a new experimental method of studying low-energy pion-nuclear interaction.

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1. The current studies of low-energy pion-nuclear interaction, which are stimulated by a number of basic problems in nuclear physics, are conducted in order to reconstruct the distribution of nucleons in the nuclei and the pion-nucleon intranuclear potential.[1] Extensive experimental data on pion-nuclear processes in synchrocyclotrons were published, for example, in a review article by Bikenshtoss.[1]

The purpose of this paper is to focus attention on the possibility of developing a new experimental method of studying low-energy pion-nuclear interaction, which is based on the angular distribution of elastically scattered isobaric nuclei of the neighboring elements predicted by:

$$X^A_Z + X^A_{Z+1} \rightarrow X^A_Z + X^A_{Z+1}, \qquad (f_c(\theta)). \tag{1}$$

The appropriate experiments can be set up now using the available multiply charged ion tandem accelerators.

2. It is known that if several channels are needed for the quantum transition, then the observed values receive an interference contribution from them. The elastic scattering of isobaric nuclei (1) is described by the coherent sum of two amplitudes, one of which corresponds to direct (potential) scattering of the nucleus of a beam at an angle θ and the other describes the scattering at an angle $(180^\circ - \theta)$ of the target nucleus that experiences a single-pion charge exchange:

$$X^A_Z + X^A_{Z+1} \rightarrow X^A_{Z+1} + X^A_Z, \qquad (f_\pi(180^\circ - \theta). \tag{2}$$

In the center-of-mass system the angular distribution of nuclei X^A_Z observed experimentaly is given by

$$\frac{dN}{d\theta} = \mathrm{const}\Big(|f_c(\theta)|^2 + |f_\pi(180^\circ - \theta)|^2 + 2\mathrm{Re}\, f_c(\theta) f^*_\pi(180^\circ - \theta)\Big). \tag{3}$$

The first two terms in Eq. (3) correspond to the classical contribution of two possible scattering mechanisms [(1) and (2)], the second of which describes their quantum interference. The angular oscillations of this term are in general possible at any collision energy E, their detection and measurement would yield maximum information at the subbarrier and near-barrier energies when the shape of the amplitude $f_c(\theta)$ is well known[3]

$$f_c(\theta) = -\frac{Z(Z+1)}{4E}\exp(i\eta \ln \epsilon^2 + i\eta), \tag{4}$$

and the amplitudes of the inelastic processes are factorized[4,5]:

$$f_\pi(\theta) = b_\pi(\theta) f_c(\theta), \qquad \eta = Z(Z+1)\sqrt{\frac{M}{2E}} >> 1. \tag{5}$$

[In Eqs. (4) and (5) $\epsilon = \sin^{-1}(\theta/2)$, η is the Coulomb quasi-classical parameter, M is the reduced nuclear mass, $b_\pi(\theta)$ is the amplitude of the probability that a pion charge exchange will occur as a result of nuclear motion along the Coulomb trajectories, and $e = h = c = 1$.] The situation is analogous to that which occurs as a result to elastic scattering of nuclei which differ by one lossely bound cluster (or nucleon).[2] A reliable clue, which facilitates identification of the predicted effect, is the characteristic "Coulomb" oscillation frequency. Near the scattering angle $\theta = 90^\circ$ $\Delta\theta = 180^\circ/\eta$.

To estimate the absolute value of the oscillations, we use the Born approximation with distorted waves[3]:

TABLE I.

X^A_Z, X^A_{Z+1}	Be^{10}, B^{10}	C^{14}, N^{14}	Ne^{22}, Na^{22}	(Ar^{40}, K^{40}) or (K^{40}, Ca^{40})
E, MeV (c.m.s.)	4.6	8.7	19.6	64.5
$\Delta\theta$, deg	55°	31°	14°	6°
b_π (90°)	0.32	0.14	0.05	0.05

$$f_\pi(\theta) = -\frac{M}{2\pi} < k_f^{(-)} | \hat{u}_\pi | k_i^{(+)} > \tag{6}$$

Here, $|k_{i,f}^{(\mp)}\rangle$ are the Coulomb wave functions with a converging and diverging spherical wave in the asymptotic form. The single-pion exchange interaction operator has a simple form[10]:

$$\hat{u}_\pi = v_0 \frac{e^{-\mu R}}{R}, \quad v_0 = \frac{3}{4} T^2 \left(\frac{\mu}{2m}\right)^2 (\mu R_0)^{-4} \exp(2\mu R_0)\,. \tag{7}$$

where μ is the pion mass, m is the nucleon mass, T is the peak of the pion-nucleon interaction in the nucleus, $g^2 = T^2/\lambda = 56\,\pi$ is the constant of the pion-nucleon interaction in the vacuum, and R_0 is the nuclear radius $[R_0 = 1.5\,(A)^{1/3}\Phi]$.

The calculation according to Eqs. (6) and (7) yields

$$b_\pi(\theta) = 2\frac{v_0}{v} e^{-\mu a} K_0(\mu a \epsilon), \tag{8}$$

where $K_0(\xi)$ is the McDonald function, $a = [Z(Z+1)/2E]$ and $E = (Mv^2/2)$. At $E = E_B \cong 0.5\, Z^2 A^{-1/3}$ MeV, in the neighborhood of the scattering angle $\theta = 90°$ according to Eq. 8, we obtain:

$$b_\pi(90°) \approx 25\lambda A^{-5/6} Z^{-1} \exp(-0.45\sqrt[3]{A}). \tag{9}$$

For $A \sim 10$ b_π (90°)$\sim$0.3. The specific examples of nuclei and parameters, which are most suitable for the measurments, are given in Table I. (The systems, which contain either the stable isotopes or the long-lived isotopes, are indicated.) Of particular interest is the unique opportunity to measure the renormalization parameter λ in the neighborhood of the double-magic Ca^{40} nucleus. If the pion peak is reconstructed extensively as a result of the "double-allowed" transition,[2] then the oscillation amplitudes of the two processes (whose other parameters are similar) will differ significantly:

$$\begin{aligned} Ar^{40} + K^{40} &\to Ar^{40} + K^{40}, \\ K^{40} + Ca^{40} &\to K^{40} + Ca^{40}. \end{aligned} \tag{10}$$

Since at $\lambda = 1$ and $E = E_B = 50$ MeV an estimate of the oscillation amplitude for processes (10) yields values of the order of several precent, thus the collision energy should be increased to 60–65 MeV in searching for the effect. (At $\theta = 90°$ these energies correspond to the conditions for grazing collisions.)

3. It should be noted that a situation similar to that described above also occurs in more complicated scattering processes accompanied by a Coulomb excitation of one of the isobaric nuclei or by nucleon (or cluster) tunneling. In particular, if two competing mechanisms (subbarrier neutron transfer[11] and pion charge exchange[10]) contribute commensurably to the reaction cross section[10,11]

$$C^{13} + N^{14} \to C^{14} + N^{13} \tag{11}$$

then the angular distribution of the N^{13} nuclei will have oscillations with a period $\Delta\theta \approx 20°$ ($E = 15$ MeV).

The ability to measure the ratios of two amplitudes of basically different processes (pion and nucleon processes) in a single experiment is the most attractive feature of these investigations.

1) The theory of pion condensation, which envisioned the possibility that the pion-nucleon interaction in the nucleus can be changed significantly, made this area of investigation very actual.[2]

2) The interference oscillations, which were predicted for this case in Ref. 6 (1967), were observed experimentally in 1968 in Zurich[7] and Heidelberg.[8] A large number of papers dealing with this matter has been published since (see, for example, Ref. 9).

3) A more precise calculation is possible, but is premature at this time.

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