Optical synchronous detection

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A method of separating amplitude and phase fluctuation spectra of a medium by means of two-beam interferometer is proposed and implemented. Possible generalization of this method to spatially-coherent fields is indicated. Separate fluctuation spectra are obtained for a liquid crystal.

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1. Information concerning processes that occur in a medium or on its surface is contained in the spectra of scattered light.[1,2] The autocorrelation function of the

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photocurrent $g_i(\tau)$ for random Gaussian processes defines the square of the modulus of the field correlation function

$$g_i(\tau) = 1 + |g_E(\tau)|^2 \qquad g_E(\tau) = \langle E^x(t)E(t+\tau)\rangle/|E|^2$$

and therefore, it fails to convey information concerning phase fluctuations.[1,3]

This observation holds for all methods of light scattering in use today, including optical heterodyning, photon counting, and, clearly, holography.

2. In this report we show that the photocurrent spectrum of the two-beam interferometer with a scattering object yields information on the complete set of its first-order spectra.

Let $\mathscr{E}_s(t) = [1+\alpha(t)]\cos[\omega_0 t + \phi(t)]$ and $\mathscr{E}_r(t) = \gamma\cos[\omega_0 t + \psi]$ be the scattered and reference waves, respectively, where $\alpha(t)$ and $\phi(t)$ are determined or random functions of time ($\alpha,\phi \ll 1$) and ψ is a constant phase difference. The dimensionless photocurrent function is as follows:

$$i(t) = [1+\alpha(t)]^2 + \gamma^2 + 2\gamma[1+\alpha(t)]\cos[\psi - \phi(t)], \tag{1}$$

where the rapidly oscillating terms containing $2\omega_0$ are omitted. An expression for the autocorrelation function may be obtained with accuracy to the second-order terms $\Phi_{\alpha\alpha} = \langle \alpha\alpha'\rangle$, $\Phi_{\phi\phi} = \langle\phi\phi'\rangle$, $\Phi_{\alpha\phi} = \langle\alpha\phi'\rangle$, where $\alpha' = \alpha(t+\tau)$, and the first-order additive constants $\langle\alpha^2\rangle$, $\langle\phi^2\rangle$, and $\langle\alpha_\phi\rangle$ which do not contribute to the spectrum, $g_i(\tau) = [1+\gamma^2+\widetilde{C}]^2 + [2+\widetilde{C}]^2\Phi_{\alpha\alpha} + S^2\Phi_{\phi\phi} + 2\widetilde{S}[2+\widetilde{C}]\Phi_{\alpha\phi}$, where

$$\widetilde{C} = 2\gamma\beta\cos\psi, \quad \widetilde{S} = 2\gamma\beta\sin\psi, \quad \beta = \exp[-\langle\phi^2\rangle/2] \tag{2}$$

and the normal distribution law is assumed to hold after averaging.[4]

It is important to take into account the contribution of shot noise $Ge\langle i(\psi)\rangle$ to the fluctuation intensity spectrum $W(\psi) = \pi^{-1}\int_0^\infty g_i(\tau)\cos\Omega\tau\, d\tau$ shot where G is the photocurrent gain.[3]

$$\begin{aligned} W(\psi) = {} & [1+\gamma^2+\widetilde{C}]^2\delta(\Omega) + [2+\widetilde{C}]^2 S_{\alpha\alpha}(\Omega) + \widetilde{S}^2 S_{\phi\phi}(\Omega) \\ & + 2\widetilde{S}[2+\widetilde{C}]S_{\alpha\phi}(\Omega) + Ge[1+\gamma^2+\widetilde{C}]. \end{aligned} \tag{3}$$

Equation (3) serves as a basis for the simple method of separating the spectra $S_{ij}(\Omega)$ $(i,j=\alpha,\phi)$ using four independent measurements of $W(\psi)$ for $\psi = \pm\pi/2, 0, \pi$.

$$S_{\alpha\alpha}(\Omega) = [W(0)+W(\pi)]/8(1+\gamma^2) - \frac{Ge}{4} = [W(0)-W(\pi)]/16\gamma\beta - \frac{Ge}{4},$$

$$S_{\phi\phi}(\Omega) = \left[W\left(+\frac{\pi}{2}\right) + W\left(-\frac{\pi}{2}\right)\right]/8\gamma^2\beta^2 - [W(0)+W(\pi)]/8\gamma^2\beta^2(1+\gamma^2) - \frac{Ge}{4\beta^2},$$

$$S_{a\phi}(\Omega) = \left[W\left(+\frac{\pi}{2}\right) - W\left(-\frac{\pi}{2}\right)\right]/16\gamma\beta.$$

In contrast with conventional methods of photodisplacement, the significant feature of the proposed method of "optical synchronous detection" (OSD) is its potential measurement for any arbitrary phase ψ.

3. The OSD method may be generalized for the spatially-coherent field

$$\mathcal{E}_n(r, t) = [1 + a_n(r, t)]\cos[\omega_0 t + \phi_n(r, t)]$$

with a common or separate reference beams $\mathcal{E}_r(r,t) = \gamma_n \cos[\omega_0 t + \psi_n(r)]$, where ψ_n is the steady-state phase difference with respect to the corresponding signal $\mathcal{E}_n(r,t)$. The reciprocal correlation function of the photocurrents of two receivers $i_n(r,t)$ and $i_m \times (r+\rho, t+\tau)$ contains information concerning intercorrelations of fluctuations $\Phi^{nm}_{aa}(\delta,\rho) = \langle \alpha_n(t,r)\alpha_m(t+\tau, r+\rho)\rangle$ and $\Phi^{nm}_{\phi\phi}(\tau,\rho) = \langle \phi_n(t,r)\phi_m(t+\tau, r+\rho)\rangle$.

$$W(\psi_n, \psi_m) = [1 + \gamma_n^2 + \widetilde{C}_n][1 + \gamma_m^2 + \widetilde{C}_m]\delta(\omega) + S^{nm}_{aa}(\Omega, \rho)[2 + \widetilde{C}_n][2 + \widetilde{C}_m]$$

$$+ S^{nm}_{\phi\phi}(\Omega, \rho)\widetilde{S}_n\widetilde{S}_m + S^{nm}_{a\phi}(\Omega,\rho)\widetilde{S}_m[2 + \widetilde{C}_n] + S^{mn}_{a\phi}(\Omega, \rho)\widetilde{S}_n[2 + C_m].$$

The fluctuation spectra $S^{nm}_{ij}(\Omega,\rho)$ may be extracted from the spectrum of the reciprocal photocurrent intensity, at point r_n and $r_m = r_n + \rho$, for example

$$S^{nm}_{a\phi}(\Omega, \rho) = \left[W\left(0, \frac{\pi}{2}\right) - W\left(0, -\frac{\pi}{2}\right)\right]/8\beta_m\gamma_m(1 + \beta_n\gamma_n);$$

$$S^{nm}_{aa}(\Omega, \rho) = W(0, 0)/4(1 + \gamma_n\beta_n)(1 + \gamma_m\beta_m).$$

We should emphasize that the differences in principle of the method under discussion and the Brown-Twiss intensity interference [method] for which the reference points are omitted and the function $g_i(\tau)$ does not contain the phase ψ.

In the technology of optical filtering and pattern recognition OSD of a correlation optical two-dimensional signal with the reference wave yields information concerning the fluctuation spectra of an object.

4. The spectral separation experiment based on Eq. (4) was carried out on the Michelson interferometer with light focusing in the measuring arm.[5]

The phase ψ of the reference beam was varied by a mirror with an intensity-calibrated piezoceramic; the photodiode current proportional to the local intensity of the interference field was amplified and passed on to an analyzer.

The absolute measurements of slow phase fluctuations were carried out by means of a method of time intervals[6] in which for a periodic displacement of the reference mirror the electronic circuit generates a voltage pulse with a duration proportional to phase of the scattered wave.

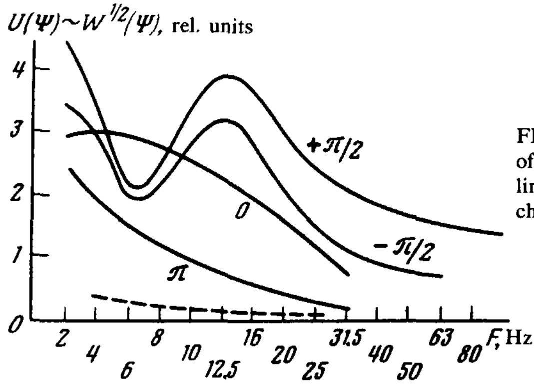

FIG. 1. Fluctuation spectra for four phase values of interfering waves $\psi = \pm \pi/2, 0, \pi$. The dotted line indicates the shot noise level, F—average channel frequency.

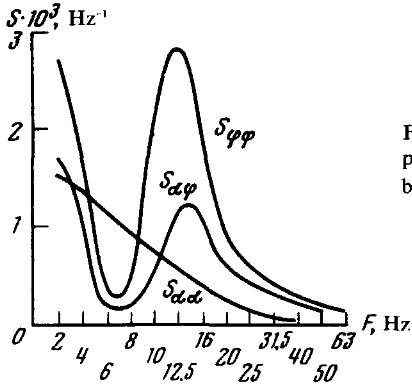

FIG. 2. Spectral density of intensities of amplitude $S_{\alpha\alpha}$, phase $S_{\phi\phi}$ and reciprocal $S_{\alpha\phi}$ fluctuations calculated on the basis of results in Fig. 1.

As an object we used a cell containing MBBA liquid crystal. The wave fluctuation spectrum was measured at a small scattering angle $\theta \leqslant 0.05$.

Figure 1 shows the fluctuation spectra obtained by the SG-1 multi-channel analyzer with the cell voltage of 3.2 V for $\phi = \pm \pi/2, 0, \pi$. The mean channel frequency is F. The shot noise spectrum is indicated by the dashed line. Figure 2 show the fluctuation spectra $S_{\alpha\alpha}(F)$, $S_{\phi\phi}(F)$ and $S_{\alpha\phi}(F)$ calculated by means of Eq. (14) for $\beta = 1$ and $\gamma = 3.75$. The absolute values of spectral density were determined by calibrating the analyzer. Intense phase fluctuations in the frequency range 0.1–2 Hz were measured independently by the method of time intervals. The second maximum at $F = 16$ Hz is clearly associated with the characteristic relaxation time for liquid crystals ($\tau_r = 0.1$–0.05 sec). A distinct correlation of the amplitude and phase fluctuations is observed at these frequencies.

In conclusion, we should point out that the proposed method may be effective in the study of fluctuations of coherent boson fields of a different nature, for example, in superconductors.

The correlation function Eq. (2) may be expressed in the general form

$$g_i^{(n)}(\tau) = \mathrm{Sp}[\hat{\rho}, (\psi, \gamma)\hat{\Phi}(\tau)],$$

where $\hat{\rho}_{ij}(\psi,\gamma)$ is the density matrix which defines the contribution to the spectrum of fluctuations $\Phi_{ij}(\tau)$.

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