Experimental investigation of beam collapse during the self-focusing in a nonlinear medium

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The nature of beam collapse in a nonlinear Kerr medium was studied. The photometrically measured absolute density distribution of energy passed through the focus was used to study the nature of the field $E\sim 1/(|z-z_f|^{\alpha})$. We obtained the values of α and compared them with theory. We show that the size of the focus may substantially differ from the size of the beam track. We show the existence and evaluate the length of the waveguide extension behind the moving focus which causes the focus to actually form a part of the waveguide.

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Formation of foci[4,5] whose sizes depend on the properties of the medium and the dynamics and dimensions of the beam, is possible in the course of self-focusing[1] (see also reviews[2,3]) i.e., self-constriction of a high-intensity beam in a nonlinear medium. However, the nature of collapse of the field $E\sim 1/(|z-z_f|^{\alpha})$ has not been heretofore studied experimentally. In addition to this, formulas derived by a number of authors[6-12] that describe features of the collapse differ substantially from each other. In our work we 1) report on the first experimental study of the nature of the collapse using as an example the energy density distribution as the beam propagates, 2) evaluate the effective collapse factor α, and 3) show the existence of a waveguide extension behind the focus.

1. Experiments were conducted using a beam from a single-mode monochromatic ruby laser with output of up to 150 kW and pulse halfwidth of 10 nsec. The near-Gaussian beam was focused by an $F = 50$-cm (case A) or $F = 10$-cm (case B) lens onto the free surface of a nonlinear medium (partially filled vertically-positioned cell[15]). We used nitrobenzene as a nonlinear medium whose depth in case A was 10 cm and in case B, 1,5 and 10 cm. The initial beam radius at the point of entry into the medium with respect to an e-fold intensity drop was $a_0 \approx 50\ \mu$m (A) and $\approx 10\ \mu$m (B); moreover, the initial divergence due to diffraction reduced the problem to the case of self-focusing of an unfocused beam.

Emission from the cell end was focused at a 10-fold amplification on a film and the exposure was measured photometrically after yet another 20-fold film image amplification. Both calibration and photometry of the exposure and the energy measurements were carried out for the same laser pulse.

In case A we compared controlled exposure with the integral of the incident radiation recorded by means of the FK-19 calibrated photocell. The recorded data were first compared with the readings of an auxiliary photocell set to record radiation reflected from a tilted glass plate, and subsequently the FK-19 was replaced by film while the beam power was varied by means of a graduated attenuator. After time integration the auxiliary photocell data were compared with the film exposure. In case B, the transmitted energy was recorded by means of a POG.

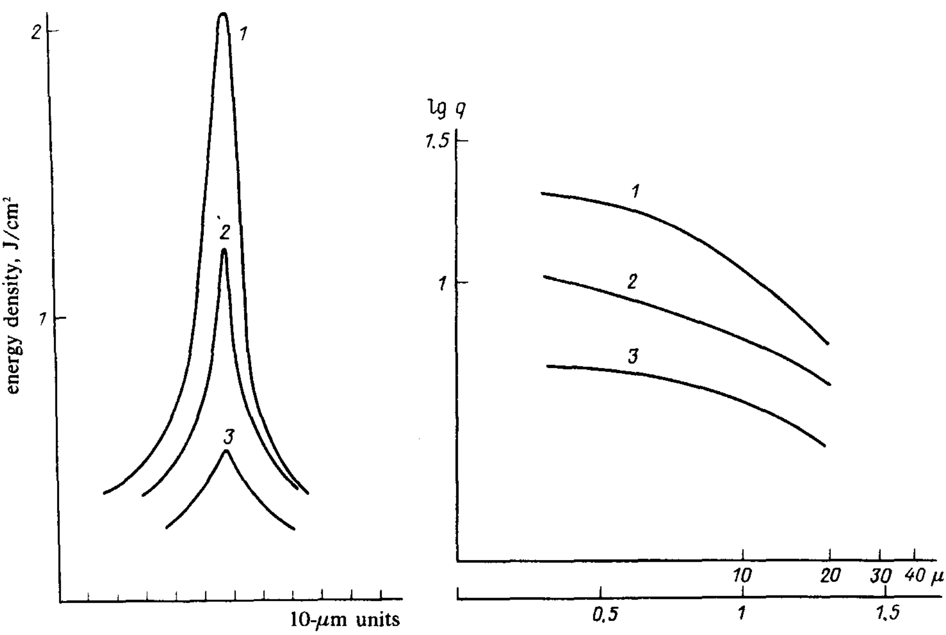

FIG. 1(A). Radial energy density distribution $q(r)$ and $\log q$ ($\log r$) for $a_0 = 50\ \mu$m, $L = 10$ cm; 1—$P = 20$ kW, 2—$P = 70$ kW, 3—$P = 130$ kW.

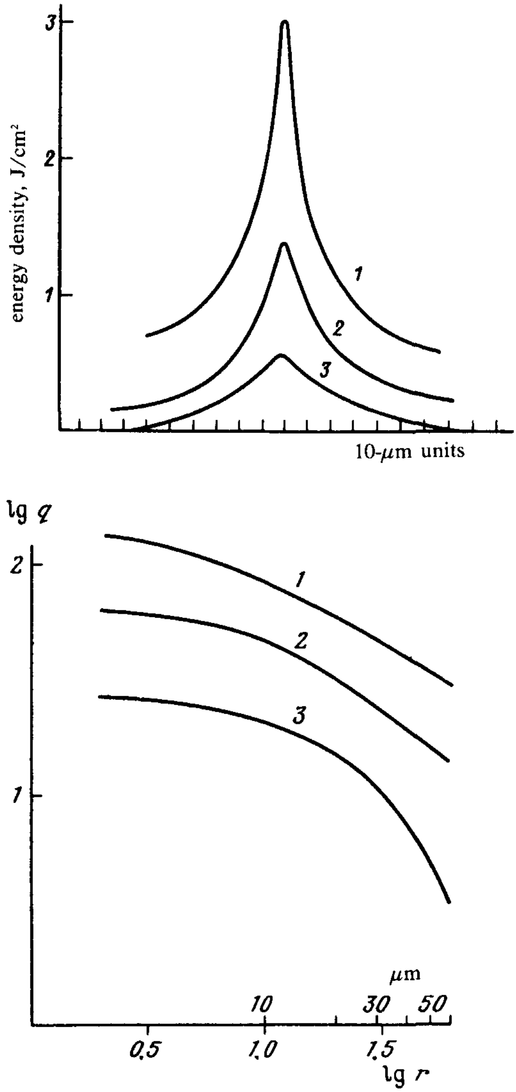

FIG. 1(B). Radial energy density distribution $q(r)$ and $\log q$ ($\log r$) for $a_0 = 10$ μm, $P = 100$ kW; 1—$L = 1$ cm, 2—$L = 5$ cm, 3—$L = 10$ cm.

Figure 1 shows the absolute energy density q(J/cm^2) as a function of radius r: case A—Fig. 1(A) and case B—Fig. 1(B). The functions are shown in the linear and twice the logarithmic scales.

2. Modeling of the beam collapse was carried out for the sake of analyzing the experimental data and determining the collapse parameters.

1. In the case of strong beam compression when $a_f \ll a_0$ (as is the case for $F = 50$ cm) $a \approx a_f(1 + |z - z_f|/\Delta)^\alpha$, where a_f is the smallest dimension in the focus. Thus, the current density $I \approx (P/\pi a^2) f^2[r/a]$, where f is a function that characterizes the radial field distribution (Gaussian-like function). Assuming that the focus moves with speed

v_f (which remains practically constant as the focus intersects the plane of observation) we obtain the following expression for energy density:

$$q \approx \int I dt \approx \int I \frac{dz_f}{v_f} \approx \frac{1}{\alpha} \frac{P}{\pi a_f} \frac{\Delta}{v_f} \left(\frac{a_f}{r} \right)^{2 - 1/\alpha} \int_0^{r/a_f} f^2(\xi) \frac{d\xi}{\xi^{1/\alpha - 1}} \; ;$$

where $\xi = r/a_f(1 + x/\Delta)^\alpha$ and $\alpha > \frac{1}{2}$. (In the case $\alpha = \frac{1}{2}$, $q \sim \ln r$, i.e., $q'_r \sim -1/r$).

The form of the function $q(r)$ and comparison with the experimental data may be used to determine both α and a_f; (a) for $r \gg a_f$, we get $q \sim 1/r^{2-1/\alpha}$, i.e., $2 - 1/\alpha = d \log q / d \log r$; (b) knowing α we may determine a_f from the condition for the decrease of the function

$$\Phi = r^{1/\alpha - 1} \frac{d}{dr} q r^{2 - 1/\alpha} = r q'_r + \left(2 - \frac{1}{\alpha} \right) q \sim f^2\left(\frac{r}{a_f} \right):$$

an e^2-fold decrease in Φ is characterized by a_f; (c) $q(r=0) = P\Delta / [\pi a_f^2 v_f (2\alpha - 1)]$ may be used to determine the parameter Δ, i.e., extent of "stretch" of focus $l_f \approx 2\Delta (e^{1/\alpha} - 1)$ [with respect to e^2-fold decrease in $I(r)$]. In this manner case A ($a_0 = 50\ \mu$m, $L = 10$ cm) results were calculated and the corresponding values of α were obtained for $P = 20$, 70 and 130 kW: $\alpha = 0.85$, 0.67 and 0.67; $a_f = 6$,12 and 12 μm and $l_f = 2.8$, 1.4 and 0.57 cm (which exceeds the Fresnel lengths by an order of magnitude).

2. In the case of weak beam compression ($a \sim a_0$) ($F = 10$ cm) the preceding extrapolation may not be applicable. Assuming, for example, that in the aberrationless case $a^2 = a_0^2 + A\,[P_{cr} - P(t)]z^2$ for $P > P_{cr}$ $a^2 = a_0^2(1 - z^2/z_f^2)$, i.e., only for $z - z_f \ll z_f$ we get $a \sim (z - z_f)^\alpha$ with $\alpha \approx \frac{1}{2}$ in this case. Since in the case B $r \gg a_0$ is studied, film exposure may be associated with a pre-focusing stage. Assuming $I = P(t)/\pi a^2(t)$ and $q = \int_0^{t_r} I dt$, where t_r is the instantaneous radiation cone passes the receiving point which is determined from the condition $a^2 = r^2 = a_0^2 + A\,[P_{cr} - P(t_r)]z^2$, and assuming for the sake of simplicity that the exposure of interest to us occurs in the quasi-linear growth portion of power $P(t) = \dot{P}(t)$ we get:

$$q \sim \left(\frac{r^2}{a_D^2} - \ln \frac{r^2 e}{a_D^2} \right)$$

where $a_D^2 = a_0^2 + z^2\theta_D^2$, a_D is the size of diffraction-broadened beam at a distance z. At $r^2/a_D^2 \ll 1$—which is known to satisfy both case A and case B—we get

$$q \sim -\ln\{ r^2 e / a_D^2 \} \quad \text{or} \quad q'_r \sim r_D / r, \ \text{i.e.,} \quad \ln|q'_r| \sim -\ln r .$$

We should note that the function holds also for $r \leqslant a_0$ since the only approximation was $r \ll a_D$. Thus, the sharp change in the slope of the function $\ln|q'_r|$ from $|\ln r|$ may be used to determine a_f which characterizes the final transverse size of the focus. Values of a_f may be used to determine the effective length of the "focus" l_f—along which the intensity falls e^2-fold—from the expression

$$q(0) = \frac{P}{\pi a_f^2} \frac{l_f}{v_f} .$$

In this manner the results of case B ($a_0 = 10\,\mu$m) were miscalculated and the following obtained: at $P = 40$ kW—$a_f = 12$ and $16\,\mu$m and $l_f = 2.5$ and 4.3 cm for $L = 5$ and 10 cm, respectively, and at $P = 100$ kW—$a_f = 6, 12$, and $30\,\mu$m, and $l_f = 0.17$, 1.0, and 3.5 cm for $L = 1$, 5, and 10 cm, respectively. All values of l_f are an order of magnitude higher than the Fresnel lengths. The length of "focus" considered as a caustic and not extension is inapplicable since in the case of caustics a small portion of the beam power is focused at each cross-section which for a given exposure and radius only further lengthens the "focus."

In the above considerations we used the minimal rates of intersection of the observation plane by the focus (in the majority of cases the foci originate inside the cell at $z_{\rm org} < L$).

The equation for q contains the full power P since in the event that only the critical power $P_{\rm cr}$ enters into the focus, the full value of q must be multiplied by the number of foci $\nu = P/P_{\rm cr}$, i.e., $q \sim P$.

The aforementioned data show that:

(a) In the majority of cases the index α is approximately 2/3;

(b) The parameter a_f is substantially smaller than $a_{q/e}$ which is determined from the e-fold decrease in q;

(c) The length of "focus" exceeds several-fold the Fresnel length l_F. This shows that the observed focus constitutes a section of a waveguide, the length of the latter l_f being determined by the relaxation of nonlinearity,[13] i.e., by a path the focus follows during the relaxation time $l \approx v_f \tau_{\rm rel}$, where $\tau_{\rm rel} = 5 \times 10^{-11}$ sec for nitrobenzene.

Theory shows the beam collapse is characterized by the power $\alpha = 1/4$,[6] 2/3[8] (apparently an incorrect conclusion), $\frac{1}{2}$[7] and 1[12], and $E \sim (|\ln x|/x)^{1/2}$,[11] where $x = z - z_f$ in dimensionless units. For the sake of easy comparison we shall rewrite the above function in the form $(|\ln x|/x)^{1/2} \sim 1/x^{\alpha(x)}$, whence we get $\alpha = \frac{1}{2} + \log|\ln x|2|\log x| \approx 2/3$ in the range of interest $x = 10^{-2}$–10^{-3}. This value ($\alpha = 2/3$) in fact fits the experimental data in case A where the beam compression is strong $a \ll a_0 \approx 50\,\mu$m (we should note that $\alpha = \alpha(x)$ and it varies from $\frac{1}{2}$ to $\frac{1}{2}$ for $x \in 10$–∞ with a weak maximum $\alpha_m = 0.7$ at $x = 10^{-e}$).

In the case B, for $L = 5$–10 cm, $a_f \approx a_0 = 10\,\mu$m, i.e., at small initial dimensions, beam collapse is practically nonexistent. Therefore, it may be affirmed that in the first experiments with the self-focusing of a beam in a liquid[14] filaments extending from the focus were in fact waveguide sections.

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