Weak interaction effect in a heavy atom

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The effect of polarization of the electron shell of a heavy atom by a weak nuclear charge on the effects of parity nonconservation is evaluated and expressed in the form of an explicit function of the nuclear charge Z. The problem reduces to amplification of the indicated effects by 10–20% (for bismuth).

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The complexity of the situation that today surrounds the problem of existence of the neutral "electron-nucleon" current calls for a reexamination of effects which further complicate the picture of a heavy atom. The polarization of filled shells of an atom by a weak nuclear charge, which alters the effective weak interaction between the nucleus and valence electron pertains to this.[1] Normally, polarization of this kind leads to screening of the nucleus. If the same held also for the weak interaction, parity nonconservation effects in a heavy atom would be suppressed and, at the least, the divergence between the local results and those of the foreign accelerator and spectroscopic experiments would be eliminated.

The polarization effect (PE) in question was recently considered by Saakyan *et al.*[1] by using accepted spectroscopic methods. The complexity of this approach (cumbersome sums over a large number of states, complex matrix elements, etc.) calls for an independent, relatively simple and useful evaluation of PE for all heavy atoms. An attempt to achieve this is made in our work by means of the Green's function method in conjunction with the quasi-classical approximation.

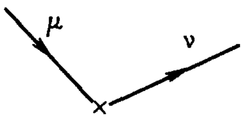

FIG. 1.

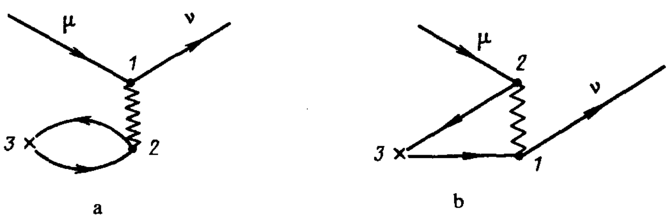

FIG. 2.

1. Addition of a state with an opposite parity to the state of the valence electron occurs due to the weak "electron-nucleus" interaction

$$W = g\vec{\sigma}(\mathbf{p}\,\delta(\mathbf{x}) + \delta(\mathbf{x})\mathbf{p}).$$

The corresponding matrix element (see Fig. 1 where the × indicates weak interaction) equals a value of the function for $x = 0$

$$W_{\mu\nu}(\mathbf{x}) = ig\sigma(\nabla\bar{\psi}_\mu(\mathbf{x})\,\psi_\nu(\mathbf{x}) - \bar{\psi}_\mu(\mathbf{x})\nabla\psi_\nu(\mathbf{x})). \quad (1)$$

Lower-order PE which plays the principal role for $Z \gg 1$ is described by the diagrams in Fig. 2: interacting weakly with the atomic shell the nucleus excites the latter producing in the process an electron and a hole; the latter annihilates with either an excited electron (direct PE, Fig. 2(a)) or with an incident electron (exchange PE, Fig. 2(b)); the annihilation occurs via the Coulomb interaction (wavy line). In the analytical sense (see Ref. 2)

$$\delta W_{\mu\nu} = -i\int d1\,d2\,d3\{\bar{\psi}_\mu(1)\,G(2,3)\,W(3)\,G(3,2) - \bar{\psi}_\mu(2)\,G(2,3)\,W(3)\,G(3,1)\}\,V_c(1,2)\,\psi_\nu(1), \quad (2)$$

where the first term corresponds to Fig. 2(a) and the second to Fig. 2(b), G is the electron Green's function, V_c is Coulomb interaction, $1 = x_1, t_1, \sigma_1$, etc.

Evidently, direct PE in the given case (in contrast to the spin-independent Coulomb interaction) is nonexistent: the virtual pairs with opposite spins introduce opposite contributions [Sp(σ) = 0, see Ref. 1]. This immediately suggests that the PE will not reduce but, instead, enhance parity nonconservation: direct PE would lead, as usual, to screening and the exchange PE contains a minus sign (see Ref. 2).

2. Proceeding with the calculation of Eq. (2)—which readily yields the original equations in Ref. 1—we shall first rewrite it in the following form:

$$\delta W_{\mu\nu} = 2ie^2 \int \frac{d\omega\, d^3k}{k^2} \int d\mathbf{x}_1 d\mathbf{x}_2 d\mathbf{x}_3$$

$$\times \bar{\psi}_\mu(\mathbf{x}_2) \exp(-i\mathbf{k}\mathbf{x}_2) G_\omega(\mathbf{x}_2,\mathbf{x}_3) W(\mathbf{x}_3) G_\omega(\mathbf{x}_3,\mathbf{x}_1) \exp(i\mathbf{k}\mathbf{x}_1)\, \psi_\nu(\mathbf{x}_1), \quad (3)$$

where

$$G_\omega(\mathbf{x}\ \mathbf{x}') = [\omega - H + i\delta(H-\mu)]^{-1} \delta(\mathbf{x}-\mathbf{x}'),$$

where $H = (p^2/2m) + U$ is the Hartree Hamiltonian and μ is Fermi energy. This yields[2)]

$$\delta W_{\mu\nu} = O W_{\mu\nu}(\mathbf{x})|_{\mathbf{x}=0}, \quad (4)$$

where

$$O = 2\pi e^2 \int_{-1}^{1} dt \int \frac{d^3k}{k^2}\, \delta\left(\mu - \frac{1}{2}(H_+ + H_-) + \frac{t}{2}(H_+ - H_-)\right),$$

where $H_\pm = [(\mathbf{p}_{\mu\nu} \pm \mathbf{k})^2/2m] + U$ and the operators $\mathbf{p}_{\mu,\nu}$ act on $\psi_{\mu,\nu}$, respectively.

3. The quantum numbers of states μ, ν and the parameter Z have large values which hopefully suggests the applicability of the quasi-classical approximation corresponding to substituting the corresponding classical momenta for the operators $\mathbf{p}_{\mu,\nu}$. It appears, however, that this yields only an order of magnitude estimate since, according to Eq. (4), the small distance region ($x \sim a_0/Z$, a_0 is Bohr's radius) is important.

The quasi-classical result is as follows

$$O = \frac{1}{\pi a_0} \int_{-1}^{1} dt\, \frac{\ln(P/\kappa)}{P}, \quad (5)$$

where

$$\mathbf{P} = \frac{1}{2}(\mathbf{p}_\mu - \mathbf{p}_\nu + t(\mathbf{p}_\mu + \mathbf{p}_\nu)), \quad \kappa^2 = m(\epsilon_\mu + \epsilon_\nu + t(\epsilon_\mu - \epsilon_\nu) - 2\mu).$$

The component integral in Eq. (5) is bounded between the values $2\ln(pa_0)/p$ ($\mathbf{p}_\mu$ and $\mathbf{p}_\nu$ are antiparallel) and $2\ln^2(pa_0)/p$ ($\mathbf{p}_\mu$ and $\mathbf{p}_\nu$ are parallel), where $p_\mu \approx p_\nu = p \sim Z/a_0$. This yields the final value

$$\ln Z/Z \lesssim \delta W_{\mu\nu}/W_{\mu\nu} \lesssim \ln^2 Z/Z, \quad (6)$$

the original result being most likely closer to the upper boundary. Numerically, for bismuth ($Z = 83$) this yields 10–20%, a figure which appears to be in a reasonable agreement with the results of Ref. 1.

It appears useful also to evaluate by means of the discussed method the PE in the hyperfine level splitting (see Ref. 3). The relationship of the type in Eq. (4) with the same value O also hold in this case. The estimate of the PE in the hyperfine splitting of the order of 10–20% is compatible with the divergence between the experimental and calculated (single-particle approximation) data.

The results given above substantiate the conclusion that the heart of the problem in the situation involving neutral currents is not associated with the theory of the parity nonconservation effect in an atom but instead lies in the purely experimental plane.

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[1)]Another effect of the same kind is the Coulomb polarization of the electron core in the matrix element of the dipole transition moment which appears as screening (see Ref. 1 where citations are made to references dealing with the problems under consideration).

[2)]Equation (4) is written in a simplified form [without auxiliary terms not having a large logarithmic factor, see Eq. (5)].

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