Onset of turbulence in rotating fluids

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(Submitted 8 January 1979)

Pis'ma Zh. Eksp. Teor. Fiz. **29**, No. 6, 329–333 (20 March 1979)

It is shown that the onset of turbulence in rotating spherical fluid layers corresponds much closer to the Ruelle-Takens model than the Landau model and differs from that observed in a cylindrical layer and in convection.

PACS numbers: 47.25.Fj, 47.30. + s

A new approach to the problem of the onset of turbulence, which was initiated by the work of Lorentz[1] and Ruelle and Takens,[2] is based on the study of the behavior of the trajectories of nonlinear dynamic systems of general topology in the phase space. Ruelle and Takens[2] showed that stochasticity in such systems develops after three or four normal bifurcations as a result of which an attracting set—a strange attractor—appears in the phase space in which all the phase trajectories are unstable. This hypothesis is an alternative of the Landau model for the onset of turbulence.[3] The current state of the problem is discussed in Refs. 4 and 5.

Experimental verification of the hypothesis,[2] based on examining the flow between the cylinders[6,7] and in a flat, convective layer,[7,8] confirmed it to some extent but also revealed significant differences which are attributable to the high degree of symmetry of this flow. It is, therefore, important to verify the hypothesis using a more general flow.

We investigated the onset of a turbulent flow in a spherical layer of fluid of thickness $(r_2 - r_1)/r_1 = 1.006$ produced as a result of rotation of an inner sphere in a wide range of Reynolds numbers $\mathrm{Re} = \Omega r_1^2/\nu$ (r_1 and r_2 are radii of the spheres, Ω is the angular velocity, and ν is the viscosity). The angular velocity was maintained to an accuracy of $\pm 0.03\%$, the fluid was thermostatically controlled to within ± 0.05 °C, $r_1 = 74.86 \pm 0.02$ mm, and the radial wobble of the shaft was ~ 0.05 mm. The film detector of the thermal anemometer was placed in the equatorial plane of the layer to match the peak of the response diagram with the direction of the radial velocity. The signal of the thermal anemometer was automatically fed to the memory bank of the BESM-6 computer. The energy spectrum of the velocity pulsations was calculated from 8192 points using a fast Fourier transform and the autocorrelation function was calculated according to the formula

$$R(\tau) = \frac{1}{T-\tau} \int_0^{T-\tau} f(t) f(t+\tau)\, dt,$$

where $\tau = 0.1\ T$ and T is $\sim$500–1000 oscillation periods at the fundamental frequency.

The main flow, which was stationary at $\mathrm{Re} < \mathrm{Re}_c^{(1)} = 460 \pm 10$, consisted of differential rotation around the axis and circulation in the meridian plane. The first

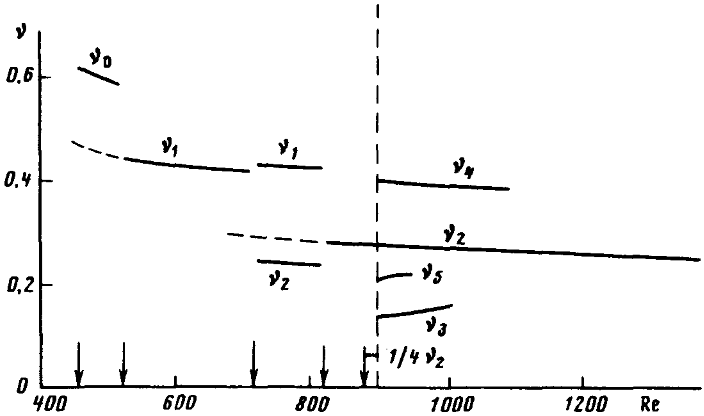

FIG. 1.

instability was accompanied by the appearance of the frequency ν_0 in the spectrum, which corresponded to the frequency of the visually observed mode with four azimuthal vortices.[9] The instrumental noise produced a damping $R(\tau)\sim 0.3\%$ (the recorded oscillations at the rotation frequency are due to the fabrication flaws of the sphere). In addition to the described mode, four more laminar modes, in which $R(\tau)$ was not dampened, were observed with increasing Re. The dependence of the dimensionless fundamental frequencies ν/Ω on the Re number for different flow regimes is shown in Fig. 1; the arrows indicate the critical Re numbers corresponding to the sequential bifurcations. The last laminar quasi-periodic mode occurred at $\mathrm{Re}=880$ when in addition to the fundamental frequency ν_2 the $1/4\ \nu_2$ subharmonic with its own harmonics appears in the spectrum (Fig. 2a); $R(\tau)$ does not dampen (Fig. 2b). Thus, in the first five modes at $\mathrm{Re}\lesssim 2\mathrm{Re}_c^{(1)}$ the flow has not more than two frequencies, excluding the rotation frequency of the sphere.

The first noticeable damping of the autocorrelation function ($\sim 2\%$) was observed at $\mathrm{Re}=895$; two satellites appeared in the spectrum near the subharmonic and its odd harmonics, which is physically equivalent to amplitude modulation (Fig. 2c). At $\mathrm{Re}=902$, three new frequencies were recorded: $\nu_3=0.1316$, $\nu_4=0.4022$, and $\nu_5=0.2078$, which apparently coincided before with the satellite $1/2\nu_2$, $3/4\nu_2$, and $3/2\nu_2$. Since the dependences of ν_2, ν_3, ν_4, and ν_5 on Re are different, we assume that all these frequencies are incommensurate. We can isolate two time intervals of damping $R(\tau)$ (Fig. 3), which correspond to fast damping $\Delta R(\tau_0)$ during the time τ_0 of the order of the period of the fundamental oscillation and to a slow damping $\Delta R(\tau_\infty)$ during the time τ_∞ of the order of 50–60 periods of the fundamental frequency.

The contribution from the ν_4 and ν_5 components increases in the spectrum with further increase of the Re number (Fig. 2d, $\mathrm{Re}=927$), but their amplitudes remain smaller than the amplitude ν_2 by about two orders of magnitude. At $\mathrm{Re}=956$ the flow spectrum, which is very complicated, is concentrated at the low frequencies in Fig. 2e. At $\mathrm{Re}=956$ the frequency ν_5 disappears and the damping $R(\tau)$ increases sharply. These two effects conceivably are related. The evolution of the spectrum at

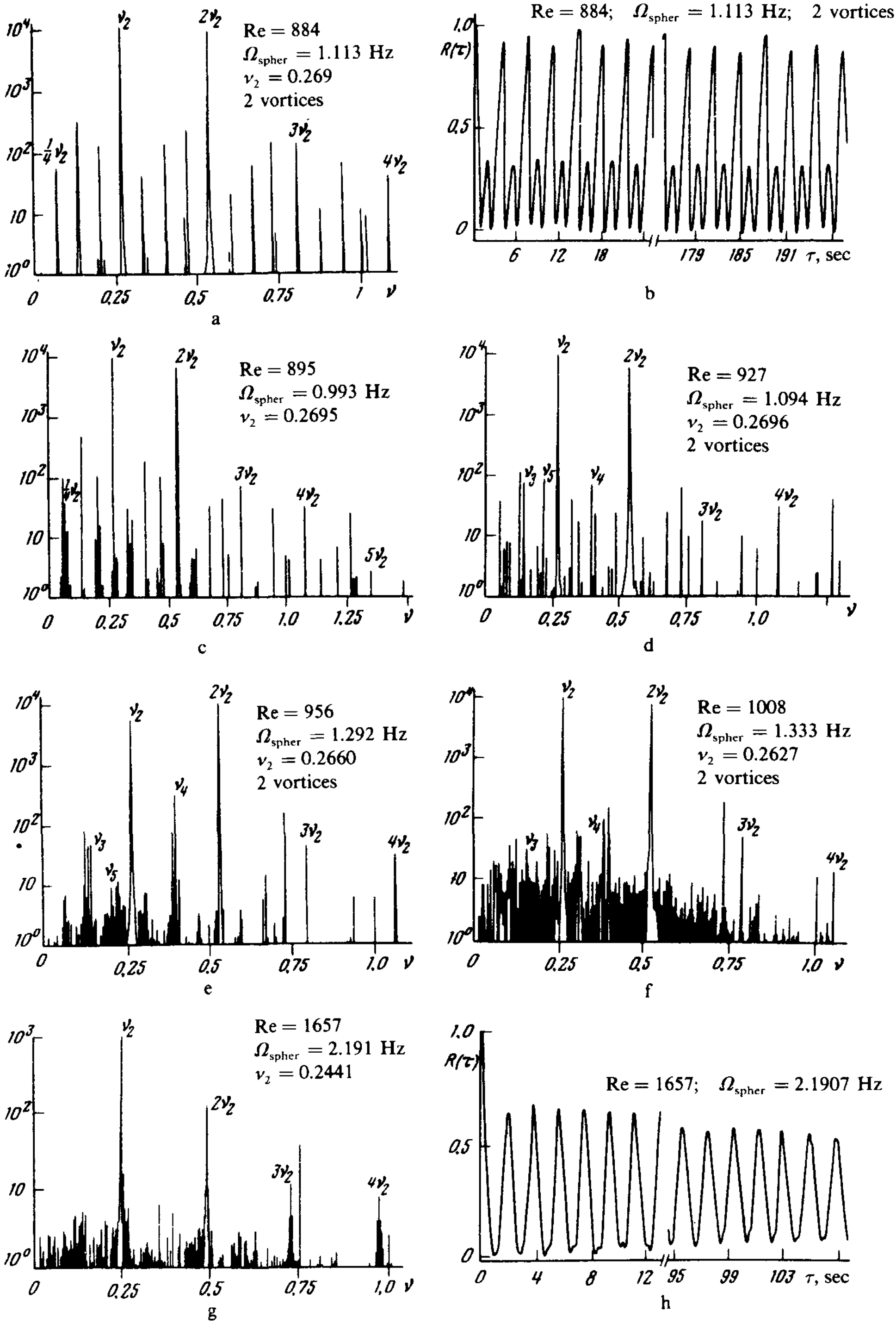
Re = 884
Ω_spher = 1.113 Hz
ν_2 = 0.269
2 vortices
a
Re = 884; Ω_spher = 1.113 Hz; 2 vortices
R(τ)
τ, sec
b
Re = 895
Ω_spher = 0.993 Hz
ν_2 = 0.2695
c
Re = 927
Ω_spher = 1.094 Hz
ν_2 = 0.2696
2 vortices
d
Re = 956
Ω_spher = 1.292 Hz
ν_2 = 0.2660
2 vortices
e
Re = 1008
Ω_spher = 1.333 Hz
ν_2 = 0.2627
2 vortices
f
Re = 1657
Ω_spher = 2.191 Hz
ν_2 = 0.2441
g
Re = 1657; Ω_spher = 2.1907 Hz
R(τ)
τ, sec
h

FIG. 2.

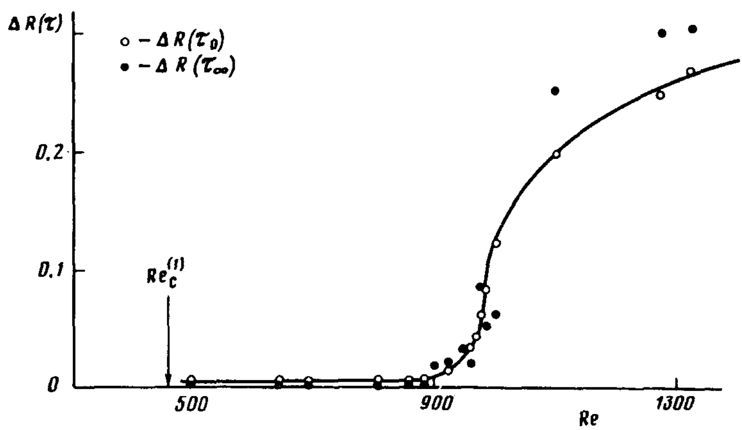

FIG. 3.

Re = 1008 is shown in Fig. 2f. At Re = 1657 the continuous component increases considerably in the oscillation spectrum (Fig. 2g), but the peaks, which exceed the noisê by ~2.5 orders of magnitude, remain at the frequency ν_2; the damping $R(\tau)$ is ~50% (Fig. 2h). The investigations were carried out to Re numbers of ~16000≈35 $Re_c^{(1)}$. At Re≈4000 the peaks at the frequency ν_2 in the background of a continuous spectrum exceed the continuous component by 1.5 orders of magnitude. The residual periodic oscillations, which contain <2% energy, were scanned even at Re~16000.

As seen from the results, the onset of turbulence in the spherical flow in thick layers is slightly different than that in Refs. 6, 7, and 8. Although the process as a whole is similar to the Ruelle and Takens model, there are also large differences: the presence of six, not necessarily normal, bifurcations before the appearance of stochasticity, the disappearance of some frequencies with increasing Re, the existence of discrete peaks in the background of the continuous spectrum. The Ruelle-Takens model[2] tends to idealize the process of transition to turbulence, but it also captures in it the main feature—the small number of bifurcations.

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