Peculiarities of recombination in a nonequilibrium plasma

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The dependence of the probability of transition between the highly excited levels of the atom on the spectral density of noise in an unstable plasma is determined. Using this dependence, we calculated the recombination coefficient under the conditions when the interaction of the electron-ion pair with the plasma noise is dominant.

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The rate of electron-ion recombination is usually estimated on the basis of the pair interaction of a recombining electron with the plasma particles. This approach is valid for a stable and slightly unstable plasma. For a highly unstable plasma the mechanism of interaction with the plasma as a whole, i.e., with the particles outside the limits of the shielding region, becomes important. In this case the collective variables can be introduced to describe the plasma and the interaction with them can be described as an interaction with a specific spectrum of elementary excitations, which is determined by the nature and degree of plasma instability.

If we regard the recombination model as a diffusion of an electron in the ϵ space of the energy terms of the atom, then the probability density of the f state of the electron in this space will obey the Fokker-Planck equation:

$$\frac{\partial f}{\partial t} = \frac{\partial}{\partial \epsilon}\left[B\left(\frac{f}{T_e} + \frac{\partial f}{\partial \epsilon}\right)\right], \tag{1}$$

where T_e is the electron temperature and

$$B = \frac{1}{2}\sum_{n'} W_{nn'}\,(\epsilon_n - \epsilon_{n'})^2. \tag{2}$$

The probability of transition $P_{nn'}$ during the time of the τ interaction, which is connected with $W_{nn'}$ by the relation $W_{nn'} = P_{nn'}/\tau$, is determined as follows

$$P_{nn'} = \frac{e^2}{\hbar^2}\int \cdots \int \psi^*_{n'}(r)\,\psi^*_{n'}(r')\,\phi(r,t)\,\phi(r',t')\,\psi_n(r)\,\psi_n(r') \times \exp[i\omega_{nn'}(t-t')]\,dr\,dr'\,dt\,dt', \tag{3}$$

where $\phi(r,t)$ is the field potential produced by all the plasma electrons at the point r of location of the recombining electron at the time t, ψ_n is the wave function of the atom

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in the energy state ϵ_n, $\omega_{nn'}$ is the transition frequency, e is the electron charge, and h is the Planck constant.

Averaging Eq. (3) over the electronic-field fluctuations, we obtain an expression for the probability $W_{nn'}$ of transition of an atom between the states nn' per unit time in the Born approximation:

$$W_{nn'} = \frac{|d_{nn'}|^2}{24\pi^3\hbar^2} \int dk \langle E^2 \rangle_{k,\,\omega_{nn'}}, \tag{4}$$

where $d_{nn'}$ is the matrix element of the dipole moment of the transition, $\langle E^2 \rangle_{k,\omega}$ is the spectral energy density of the electric-field fluctuations in the plasma, and k is the wave vector.

We shall limit our examination to a plasma in which an electron beam with a density n'_{e0}, velocity u, and temperature T'_e is propagated. In such a system near the stability boundary[1]:

$$\langle \vec{E}^2 \rangle_{k,\,\omega} = \frac{16\sqrt{2\pi}\,\pi^2}{k^2|\epsilon_e|^2} \left\{ \frac{n_{eo}}{kv_e} \exp\left(-\frac{\omega^2}{2k^2v_e^2}\right) + \frac{n'_{eo}}{kv'_e} \exp\left[\frac{(\omega - ku)^2}{2k^2 v_e'^2}\right] \right\}, \tag{5}$$

where $v_e = (T_e/m)^{1/2}$ is the thermal velocity of the plasma electrons, $v'_e = (T'_e/m)^{1/2}$ is the thermal velocity of the beam electrons, n_{e0} is the plasma density, m is the electron mass, and ϵ_e is the longitudinal dielectric permeability of the plasma-beam system.

In calculating $B(\epsilon)$ we assumed that for the highly excited states of the atom[2]

$$|d_{nn'}|^2 = \frac{e^2\hbar}{m\omega_{nn'}} \frac{32}{3\pi\sqrt{3}} \frac{1}{n^5} \frac{1}{n'^3} \left(\frac{1}{n'^2} - \frac{1}{n^2}\right)^{-3},$$

and in the transparent region of the plasma:

$$a = \left[\left(2\pi m T_e\right)^{3/2} \int_{-\infty}^{0} e^{\epsilon/T_e} \frac{\partial\epsilon}{B(\epsilon)\,\rho(\epsilon)} \right]^{-1}, \tag{6}$$

where $\delta(x)$ is the Dirac delta function, $\epsilon_l = \epsilon' + i\epsilon''$, and $\epsilon'(\omega_i) = 0$. In the frequency region $(\omega/kv_e) \ll 1$:

$$\epsilon_l = 1 - \frac{\omega_p^2}{\omega^2} + i\sqrt{\frac{\pi}{2}} \left[\frac{1}{k^2a^2} \frac{\omega}{kv_e} \exp\left(-\frac{\omega^2}{2k^2v_e^2}\right) + \frac{1}{k^2a'^2} \frac{\omega - ku}{kv'_e} \right],$$

where a is the Debye radius of the plasma and a' is the Debye radius of the plasma and the beam.

This expression is valid if

$$\frac{|\omega - ku|}{kv'_e} < 1$$

and if the thermal motion of the ions is ignored.[1] Let us integrate over n' in Eq. (2). The next integration over k can be broken down into two integrations: the first is $\int_0^{1/a} dk$, where the interaction is appreciable with the collective excitations whose energy spectrum is determined by the electron beam and the second integration $\int_{1/a}^{k_m} dk$ is inside the Debye domain where $\epsilon_l \approx 1$; k_m is determined by the minimum impact parameters. The last integration is carried out exactly and yields the value

$$B_2 = \frac{8\sqrt{2\pi}}{3}\,\frac{n_{oe} e^4 \lambda}{m v_e}\,\epsilon$$

(λ is the Coulomb logarithm) obtained by Abramov and Smirnov.[3] The integral in the first region can be approximated, taking into account the fact that the main contribution to the integrand gives the interval

$$\Delta k \sim \frac{1}{a}\,\frac{v_e}{u}\left(\frac{T'_e}{T_e}\right)^{3/2}\frac{n_{oe}}{n'_{oe}}\exp\left(-\frac{1}{2k^2 a^2}\right)$$

near $k \sim \omega_p/u$; the angle between k and u is small.

Thus, we obtain

$$B_1 = \frac{32\sqrt{2}}{9\sqrt{3}\pi}\,\frac{T'_e}{T_e}\left(\frac{v_e}{u}\right)^3\frac{|\epsilon|^{5/2}}{a\sqrt{m}}. \qquad (7)$$

It follows from Eq. (7) that $B_1 > B_2$ when

$$\epsilon/T_e > \left(\frac{u}{v_e}\right)^2\left(\frac{T_e}{T'_e}\right)^{2/3}\left(\frac{3\sqrt{3}\,\lambda}{8\sqrt{\pi}}\right)^{2/3}\frac{e^2 n_{oe}^{1/3}}{T_e},$$

which corresponds to the crucial contribution from the collective processes.

The recombination coefficient a can be calculated under these conditions by using the relation[4]:

$$a = \left[(2\pi m T_e)^{3/2}\int_{-\infty}^{0} e^{\epsilon/T_e}\,\frac{\partial\epsilon}{B(\epsilon)\,\rho(\epsilon)}\right]^{-1},$$

$$\rho(\epsilon) = \frac{2\pi^3 e^6 m^{3/2}}{\sqrt{3}\,|\epsilon|^{5/2}},$$

which was obtained when the end-point energy ϵ_0 of the excited atom, at which with overwhelming probability begins the radiation transition, satisfies the condition $\epsilon_0 > T_e$. The calculations yield

$$a = \frac{16\sqrt{2\pi}}{9\sqrt{3}} \frac{e^6 T_e^{-5/2}}{a\sqrt{m}} \left(\frac{v_e}{u}\right)^3 \frac{T_e'}{T_e}. \tag{8}$$

The absence of a contribution from the beam to the dispersion indicates that $a'/a \gg 1$ and the integration in Eq. (4) is carried out at

$$\frac{n_{oe}'}{n_{oe}} >> \left(\frac{T_e'}{T_e}\right)^{3/2} \exp\left(-\frac{1}{2k^2 a^2}\right).$$

This investigation shows that the three-particle recombination in a highly unstable plasma has to be described by taking into account the interaction not only with the electrons in the shielding region but also beyond it, i.e., with the plasma as a whole. It is important to do this in the analysis of the "motion" of an electron in the high states of the atom, where its spectrum is assumed to be almost continuous.

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