which was obtained when the end-point energy ϵ_0 of the excited atom, at which with overwhelming probability begins the radiation transition, satisfies the condition $\epsilon_0 > T_e$. The calculations yield

$$a = \frac{16\sqrt{2\pi}}{9\sqrt{3}} \frac{e^6 T_e^{-5/2}}{a\sqrt{m}} \left(\frac{v_e}{u}\right)^3 \frac{T_e'}{T_e} . \quad (8)$$

The absence of a contribution from the beam to the dispersion indicates that $a'/a \gg 1$ and the integration in Eq. (4) is carried out at

$$\frac{n_{oe}'}{n_{oe}} >> \left(\frac{T_e'}{T_e}\right)^{3/2} \exp\left(-\frac{1}{2k^2a^2}\right).$$

This investigation shows that the three-particle recombination in a highly unstable plasma has to be described by taking into account the interaction not only with the electrons in the shielding region but also beyond it, i.e., with the plasma as a whole. It is important to do this in the analysis of the "motion" of an electron in the high states of the atom, where its spectrum is assumed to be almost continuous.

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Resonance scattering of fast electrons in a single crystal

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It is shown that resonance scattering of fast ($pL > > pR > > 1$, p is the particle momentum, L is the longitudinal dimension of the potential, and R is its transverse dimension) charged particles occurs in the extended potential. The data on small-angle scattering of fast electrons in the crystal are interpreted on the basis of the examined effect.

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It is generally assumed that resonance scattering in elastic collisions applies to slow particles $pR \ll 1$.[1] As will be shown below, however, resonances in the elastic scattering can also be present in fast particles, i.e., when $pR \gg 1$, if the particle is

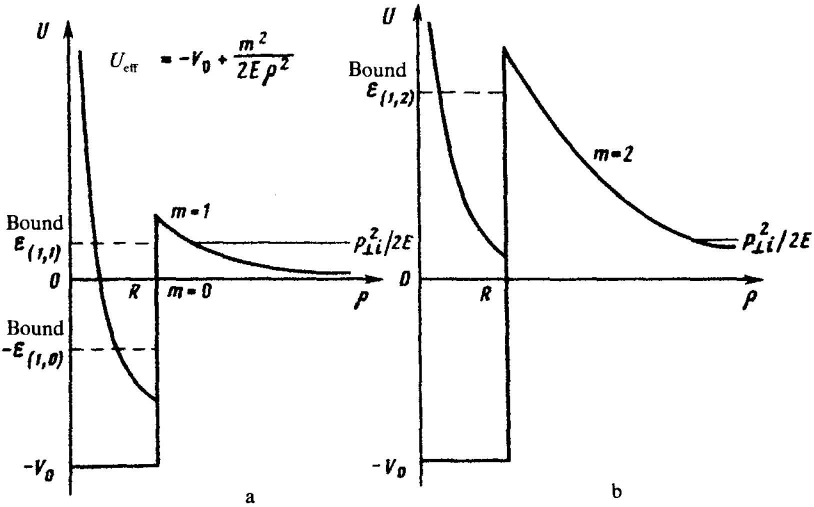

FIG. 1. Transverse cross section of the string's potential. The effective potential and bound states for the partial waves with an azimuthal momentum: a, $m = 0.1$ and b, $m = 2$.

scattered by an extended, attracting potential. Physically, this effect consists of the following: the cross section of the extended, attracting potential has bound and quasi-bound states and hence the transverse component of the incident wave of a fast particle may undergo resonance scattering on these states. In this case "the slow" transverse motion of the particle is a necessary requirement, i.e., $p_\perp R \ll 1$. This imposes a constraint on the entrance angle of the particle $\theta_0 \ll 1/pR$.

Let us examine the scattering by a string potential which has the appearance of a square well (Fig. 1):

$$U(\vec{\rho}, z) = \begin{cases} -V_0 & \text{for } \rho < R, \quad 0 < z < L \\ 0 & \text{for } \rho > R \quad z < 0,\ z > L \end{cases} \tag{1}$$

where L is the length of the string and $2R$ is its transverse diameter ($L \gg 2R$). If we assume that a fast particle $pL \gg pR \gg 1$ enters almost parallel to the z axis at an angle θ_0, then from the condition for joining the wave function of a particle at the boundary $z = 0$ and $z = L$ and for the asymptotic behavior at infinity we obtain the scattering amplitude[2]

$$f(\theta, \phi) = \frac{p}{2\pi i} \Sigma Q_{p_{\perp i}}(a, m)\, Q^*_{p_{\perp f}}(a, m) \left[\exp i \frac{a - p_{\perp f}^2}{2p} L - 1\right], \tag{2}$$

where

$$Q_{p_\perp}(a, m) = \int d^2\vec{\rho}\, Z_{a,m}(\vec{\rho}) \exp(i p_\perp \vec{\rho})$$

is the amplitude of the transition from the state of the plane wave with the momentum $p_\perp$ to the eigenstate $Z_{\alpha,m}(\boldsymbol{\rho})$ of the transverse motion in the potential (1); the summation is carried out over the compound states with the transverse energy $\epsilon(n,m) = \alpha/2p$ and momentum m. To calculate the total cross section we use the optical theorem $\sigma = (4\pi/p)\,\mathrm{Im} f(\theta,\phi_0)$. We obtain

$$\sigma = 4\Sigma\,|\,Q_{p_{\perp i}}(\alpha,\, m)|^2 \sin^2 \frac{\alpha - p_{\perp i}^2}{4p} L\ . \tag{3}$$

It was shown earlier[3] that, as a result of scattering of a fast particle by a sufficiently extended potential ($L \gg pR^2$), the effective scattering angle becomes very small $\theta_{\rm eff} \sim 1/\sqrt{pL}$ and the corresponding effective impact parameter very large $\rho_{\rm eff} \sim 1/p\theta_{\rm eff} \sim \sqrt{L/p}$. The particle with the azimuthal momentum m passes the string at a distance of $\sim m/p_{\perp i}$ and hence is scattered by the potential (1) if this distance is smaller than the effective impact parameter

$$\frac{m}{p_{\perp i}} \lesssim \rho_{\rm eff} \simeq \sqrt{\frac{L}{p}}\ . \tag{4}$$

If

$$\frac{1}{p_{\perp i}} > \sqrt{\frac{L}{p}}\ \left(\theta_0 < \frac{1}{\sqrt{pL}}\right),$$

then only the wave with $m = 0$ is scattered. In this case the scattering cross section (3), which is integrated and summed over the intermediate states, has the following form:

$$\sigma = \sigma_0 + \frac{2}{\pi}\Sigma \frac{|\epsilon^{b}(n,\, 0)|}{p}\ \frac{\sin^2\left[|\epsilon^{b}(n,\, 0)| + \dfrac{p_{\perp i}^2}{2p}\right]\dfrac{L}{2}}{\left[|\epsilon^{b}(n,\, 0)|^2 + \dfrac{p_{\perp i}^2}{2p}\right]^2} \tag{5}$$

where $\sigma_0 \sim (2/\pi)\, L/p$ is the scattering cross section from the continuous spectrum with $m = 0$, which was calculated elsewhere,[3] and the second term is determined by the bound states $[\epsilon^b(n,0) < 0]$ in the transverse potential of the attracting string. We can easily see that the main contribution to the scattering cross section is introduced by bound states which are removed a distance of $\sim 1/L$ from the edge of the well. At the same time, the scattering cross section has a narrow peak $\sim 1/\sqrt{pL}$ in width near the zero angle of entry.

Let us assume now that the angle of entry is $1/\sqrt{pL} < \theta_0 \ll 1/pR$. The effective number of waves participating in the scattering in this case is determined by the inequality (4). It follows from relation (5) that the bound states $[\epsilon^b(n,m) < 0]$ can be disregarded. However, for partial waves with $m > 0$ because of the presence of a centrifugal barrier $m^2/2ER^2$ quasi-bound states with $\epsilon^b(n,m)$ are developed in the effective potential (Fig. 1). Assuming that the barrier is infinitely high and substituting the square well for the effective well, we obtain

$$\epsilon^b(n, m) = \frac{\pi^2}{2ER^2} n^2 + \frac{m^2}{2ER^2} - V_o . \tag{6}$$

Taking into account the finite depth of the effective well ($\sim V_0$), we can calculate the number of quasi-bound states corresponding to the azimuthal momentum m

$$n^2_{max} = 2EV_o R^2 / \pi^2 . \tag{7}$$

It can be seen that $n_{\max}$, which is independent of m, is determined by the parameters of the well and by the total energy of the particle. In the energy region $E < \pi^2/2R^2V_0$ (for silicon $E \lesssim 20$ MeV) there is only one quasi-bound state $\epsilon^b(1,m)$ for a given m. It follows from Eq. (6) that $\epsilon^b(1,m) > 0$. Moreover, we always have $\epsilon(n,m) > \epsilon(n,m-1)$ (Fig. 1b). If the transverse energy now coincides with the energy of a certain quasi-bound state (Fig. 1a)

$$\frac{p^2_{\perp i}}{2E} = \frac{\pi^2}{2ER^2} n^2 + \frac{m^2}{2ER^2} - V_o , \tag{8}$$

then we have resonance scattering. The scattering cross section, which can be determined from expression (3) by integrating over the continuous spectrum from 0 to ∞, in this case has the Breit-Wigner form

$$\sigma = \sigma_o + \frac{2}{\pi} \frac{L}{p} \frac{\frac{1}{4}\Gamma^2}{(\theta_o - \theta_{o\,res})^2 + {}^1\!/_4\Gamma^2} , \tag{9}$$

where σ_0 is the cross section for scattering at some distance from the resonance. We can easily obtain $\theta_{0_{res}}$ from relation (8)

$$\theta_{o\,res} = \left(\frac{\pi^2 n^2 + m^2}{E^2 R^2} - 2\frac{V_o}{E} \right)^{1/2} . \tag{10}$$

Determination of the finite centrifugal barrier using the standard quasiclassical treatment[11] yields the following angular width of the resonance

$$\Gamma \sim \theta_{o\,res} \exp - | 2EV_o R^2 - \pi^2 n^2 |^{1/2} . \tag{11}$$

If the angle of entry remains constant and the energy of the particle is varied, then from Eqs. (8), (9), and (11) we obtain

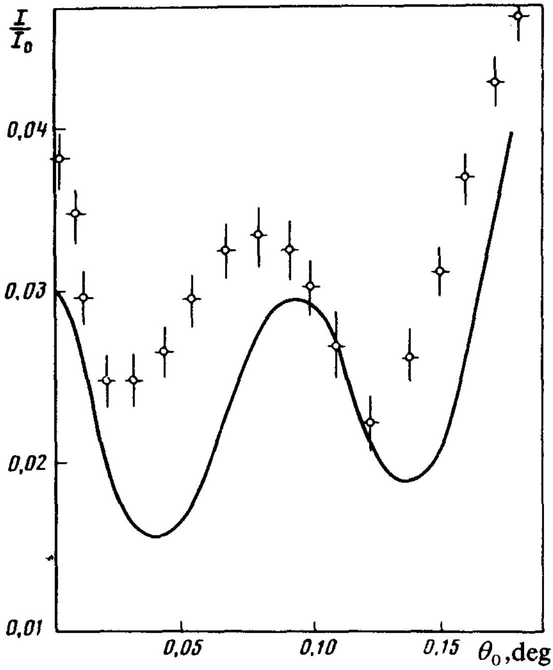

FIG. 2. The intensity of the 15-MeV electrons scattered in the 1.4-μm-thick silicon crystal as a function of the angle of entry θ_0 relative to the direction of the $\langle 111\rangle$ axis. I_0 is the intensity of the incident particles. The experimental points were taken from Ref. 4; the solid curve denotes theoretical calculation.

$$\sigma = \sigma_0 + \frac{2}{\pi}\frac{L}{p}\frac{\frac{1}{4}\Gamma^2}{(E - E_{res})^2 + \frac{1}{4}\Gamma^2}\;;\quad \Gamma \sim E_{res}\exp - |2E_{res}V_0R^2 - \pi^2 n^2|^{1/2};$$

$$E_{res} = \theta_0^{-2}\{2V_0 + [4V_0^2 + 4\theta_0^2 R^{-2}(\pi^2 n^2 + m^2)]^{1/2}\}. \tag{12}$$

The indicated resonance can be observed in the crystal when the fast particle enters it at a small angle to the crystallographic axis. In this case the longitudinal momentum transfer becomes very small $q_{\parallel} \sim 1/pR^2$ and the wave function of the particle, which is insensitive to the details of the behavior of the potential at approximately the interatomic distances, is determined by a certain potential averaged over the length of the chain.[2] Schiebel and Worm[4] observed the scattering of 15-MeV electrons along the $\langle 111\rangle$ axis of a 1.4-μm-thick Si crystal. Figure 2 shows a small-angle dependence of the intensity of the scattered particles on the angle of entry relative to the crystallographic $\langle 111\rangle$ axis. This dependence cannot be explained in terms of the usual two-wave diffraction theory and the classical string scattering.[5] From the viewpoint of the discussion conducted above, the very narrow peak near the zero angle of entry is due to the true bound states; at the same time, its width $\sim 1/\sqrt{pL} \approx 0.02°$ is very close