The form of rapidly-moving electron-hole drops

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We show that as a consequence of interaction with the lattice an electron-hole drop moving in a semiconductor should collapse without limit when its speed approaches the speed of sound.

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Electron-hole drops (EHD), deform the semiconductor around them. This interaction—described in terms of a deformation potential approximation—leads to the following effects; attraction of EHD to the crystal surface;[1] loading of a moving EHD;[2] sharp radiation braking in the case of drops crossing the sound barrier.[2,3] A moving EHD undergoes an opposite effect from the lattice[3] at low velocities $V \ll S$ (S is speed of sound in a semiconductor) EHD should shrink slightly in the direction of motion. As we shall show below, an EHD should collapse without limit as it approaches the sound barrier.

For the sake of simplicity we shall consider a semiconductor to be an isotropic medium. We shall also neglect the effect of phonon wind on EHD shape (as was shown in Ref. 4, this is acceptable provided the EHD radius is relatively small, $R_0 \lesssim 10^{-4}$ cm in Ge). In the isotropic approximation EHD interacts only with the longitudinal field rot $\mathbf{u} = 0$ [where $\mathbf{u}(\mathbf{r},t)$ is the displacement of an atom at a point $(\mathbf{r},t)$ from equilibrium]. Therefore, we shall write the following lattice and EHD Lagrangian

$$L = L_{\text{EHD}} + \int \left[\frac{\rho}{2} \left(\frac{\partial \mathbf{u}}{\partial t} \right)^2 - \frac{\rho S^2}{2} (\operatorname{div} \mathbf{u})^2 - Dn \operatorname{div} \mathbf{u} \right] d^3 r, \tag{1}$$

where L_{EHD} is the free EHD Lagrangian, ρ is the crystal density, S is the speed of longitudinal sound, D is the deformation potential constant and $n(\mathbf{r},t)$ is the carrier density in EHD.

We shall assume the EHD velocity $\mathbf{V}$ to be fixed and the electron-hole liquid (EHL) contained in it to be incompressible with a density n_0. The latter approximation is applicable if the EHD-lattice interaction is weak in comparison with the EHL binding energy (a condition highly satisfiable in Ge-type semiconductors). In this case, Eq. (1) represents a functional of derivatives of the deforming field and drop shape. The condition of least action for Eq. (1) yields equations for determining the field $\mathbf{u}$ and drop shape. The latter, as may be shown, expresses an equality at the EHD surface between the internal pressure and pressure due to surface tension forces. However, such an equality is extremely complex. To obtain a qualitative picture we shall assume that a drop has the shape of an ellipsoid of revolution (around the direction of motion, in accordance with the axial symmetry of the problem) with a volume $\frac{4}{3}\pi R_0^3 = \frac{4}{3}\pi a^2 b$, where R_0 is the mean EHD radius and a and b are semiaxes of the generating ellipse across and along the axis of rotation. In view of the fixed volume this class of surfaces will be singly-parametric (the ratio of semiaxes $\nu = a/b$ is a convenient parameter) with the surface equation $|\mathbf{r}| = R_\nu(\theta)$, where θ is the polar angle with respect to direction $\mathbf{V}$. Thus, the density n in Eq. (1) may be expressed as follows:

$$n(\mathbf{r}, t) = n_0 \Theta(R_\nu(\theta) - |\mathbf{r} - \mathbf{V}t|), \quad \text{where } \Theta(x) = \begin{cases} 1, & x > 0 \\ 0, & x < 0 \end{cases}. \tag{2}$$

The Lagrange equation for $\mathbf{u}$ has the following form:

$$\left(\Delta - \frac{1}{S^2}\frac{\partial^2}{\partial t^2}\right)\mathbf{u} = -\frac{Dn_0}{\rho S^2}\vec{\nabla}\,\Theta(R_\nu(\theta) - |\mathbf{r} - \mathbf{V}t|). \tag{3}$$

If we substitute the solution of Eq. (3) into Eq. (1), the action for Eq. (1) will become a function of ν. It can be readily shown that as a consequence of Eq. (3), the problem of least action is equivalent to the problem of least action for Eq. (1) for independent ν and $\mathbf{u}$. Direct calculations show that such a problem is in turn equivalent to a problem of finding in terms of ν the sum of the potential energy of a free EHD and one-half its energy of interaction with the lattice. If forces which cause EHD motion and EHD and lattice friction are of the same kind within a drop, the surface

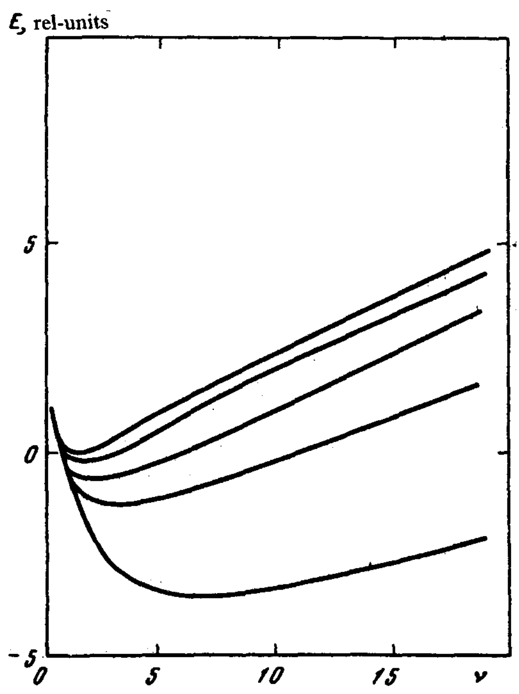

FIG. 1. The dependence of E on the ratio of semiaxes ν of drops with radius $R_0 = 6R_c$ for different mobility rates (curves from top to bottom: $\beta = 0.3$; 0.5; 0.6; 0.7; 0.8; 0.9).

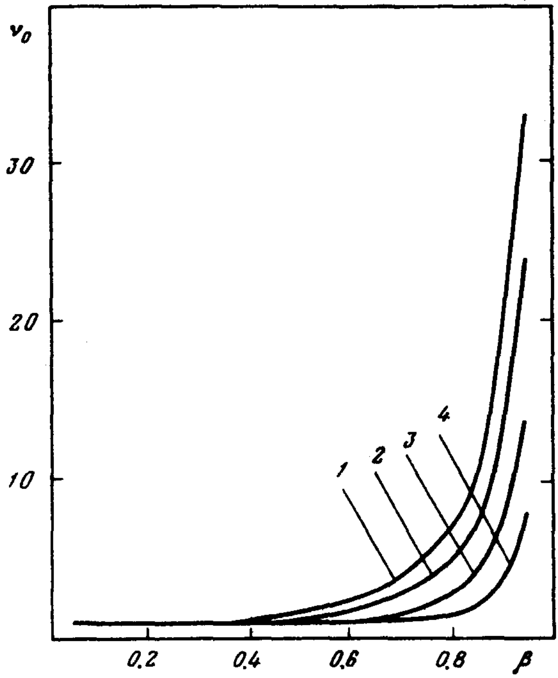

FIG. 2. The dependence of the ratio of semiaxes ν on the speed $\beta = V/S$ for drops of different mean radius: 1–$R_0 = 3R_c$; 2–$R_0 = 6R_6$ 3–$R_0 = 9R_c$; 4–$R_0 = 12R_c$.

energy is the only part of the potential energy which is shape-dependent in the steady state. The equilibrium value of ν_0 is subsequently determined from a condition on the energy minimum

$$E = aS_{\text{EHD}} + \frac{Dn_0}{2} \int \Theta(R_\nu(\theta) - |\mathbf{r} - \mathbf{v}t|) \operatorname{div} \mathbf{u}_\nu \, d^3r, \tag{4}$$

where a is the coefficient of surface tension, $\mathbf{u}_\nu$ is a solution of Eq. (3) for a given ν and S_{EHD} is EHD surface area.

The integral in Eq. (4) is calculated by means of the Fourier transformations in Eqs. (3) and (4). We also use the known formula for the surface area of an ellipsoid of revolution. Thus,

$$\frac{E}{2\pi R_0^2 a} = \nu^{2/3} + \nu^{-1/3}F_1(\nu) + \frac{\gamma}{3} \frac{\nu^2 - 1 - \beta^2\nu^2F_2(\beta, \nu)}{\nu^2 - 1 - \beta^2\nu^2}, \tag{5}$$

where

$$F_1 = \begin{cases} \arcsin\sqrt{1-\nu^2}\Big/\sqrt{1-\nu^2} & 0 < \nu < 1, \\ \ln\dfrac{\nu + \sqrt{\nu^2 - 1}}{\nu - \sqrt{\nu^2 - 1}} \Big/ 2\sqrt{\nu^2 - 1}, & 1 < \nu, \end{cases}$$

$$F_2 = \begin{cases} \ln \dfrac{1+\sqrt{(\beta^2-1)\nu^2+1}}{1-\sqrt{(\beta^2-1)\nu^2+1}} \Big/ 2\sqrt{(\beta^2-1)\nu^2-1}, & 0<\nu<\dfrac{1}{\sqrt{1-\beta^2}}, \\ \operatorname{arc\,tg}\sqrt{(1-\beta^2)\nu^2 \quad 1} \Big/ \sqrt{(1-\beta^2)\nu^2-1}, & \dfrac{1}{\sqrt{1-\beta^2}}<\nu. \end{cases}$$

$\gamma = R_0/R_c$, $R_c = a\rho s^2/(Dn_0)^2$. in Ge $R_c \cong 10^{-4}$ cm.

Figure 1 shows the typical behavior of E as a function of ν for different drop speeds. As the speed increases, the position of the maximum is displaced in the direction of larger ν. Results of the numerical calculations of ν_0 as a function of speed for drops of different sizes are shown in Fig. 2 Equation (5) may be used to obtain the asymptotic behavior of ν_0 for $\beta \to 1$, namely $\nu_0 \simeq (\frac{1}{4}\pi\gamma\beta^2)^{3/7}(1-\beta^2)^{-9/14}$. As EHD approaches the sound barrier, it should collapse without limit.

It should be noted that as $\beta \to 1$, the energy $E \sim -(1-\beta^2)^{-1} \to \infty$. Thus, at high ($V \cong S$) speeds the condition for the applicability of the incompressibility approximation is violated. Equation (2) is also known to be not applicable when the thickness of a collapsed EHD is comparable to the washing-out of its boundaries $\sim 10^{-6}$ cm. Finally, instability may develop in a collapsed EHD (due to, say, phonon wind) which may cause its break-up. In any case, the question whether an EHD may reach the speed of sound is highly problematic in the light of available results.

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