# The structure of divergences in supergravitation

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We determined the most general structure of the gauge-invariant local divergences in supergravitation and showed that both they and their contribution to the quantum equation of motion vanish in the self-dual instanton field.

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Kallosh[1] examined the theory of supergravitation in the field of a self-dual instanton. It was found that in such a field all divergences of the supergravitation (except for the trivial, single-loop divergences associated with the topological invariants[2]) vanish. Christensen and Duff[3] obtained independently and concurrently an analogous result by using a different method.

In this paper, we give the result of a detailed analysis of local invariants of supergravitation, which, first, is of interest in itself, second, makes more understandable the result of Ref. 1, and third, shows that all the gauge-invariant local structures (specifically, the counterterms) and their first field derivatives vanish in the self-dual field.

1. Let us examine the generating functional of supergravitation in the external field, which satisfies the classical equations of motion. In analyzing the divergences of this expression, we use the linearized approximation of the supergravitation,[4-6] which is sufficient[5] for determining the counterterms that vanish on the mass shell. All the local invariants in this case are constructed from the spinor superfields[6] 1) $W_{\alpha\beta\gamma}, \widetilde{W}_{\dot\alpha\dot\beta\dot\gamma}$ and from their supercovariant derivatives. These fields, which are a supersymmetrical generalization of the curvature tensor, are chiral fields, i.e.,

$$\widetilde{D}_{\dot\tau} W_{\alpha\beta\gamma} = D_{\tau} \widetilde{W}_{\dot\alpha\dot\beta\dot\gamma} = 0. \tag{1}$$

Moreover, on the mass shell they satisfy the relations[6]

$$D^{\alpha} W_{\alpha\beta\gamma} = \widetilde{D}^{\dot\alpha} \widetilde{W}_{\dot\alpha\dot\beta\dot\gamma} = 0 \,. \tag{2}$$

Using Eq. (2) we can show that the superfields $D_\alpha W_{\beta\gamma\delta}$ and $\tilde{D}_{\dot\alpha} \widetilde{W}_{\dot\beta\dot\gamma\dot\delta}$ are also chiral fields and

$$D_{\lambda} (D_{\alpha} W_{\beta\gamma\delta}) = \widetilde{D}_{\dot\lambda} (\widetilde{D}_{\dot\alpha} \widetilde{W}_{\dot\beta\dot\gamma\dot\delta}) = 0 \,, \tag{3}$$

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i.e., the $W$ and $\widetilde{D}\widetilde{W}$ fields are left handed and $\widetilde{W}$ and $DW$ are right handed. Using the well-known anticommutators of the supercovariant derivatives $D_\alpha$ and $D_{\dot\alpha}$ and introducing the notations

$$\begin{aligned} \partial_{\alpha_1\dot\alpha_1}\cdots\partial_{\alpha_k\dot\alpha_k}W_{\alpha\beta\gamma} &\equiv W_{(k)} \\ && k,\ l=0,1,\ldots \\ \partial_{\alpha_1\dot\alpha_1}\cdots\partial_{\alpha_l\dot\alpha_l}W_{\dot\alpha\dot\beta\dot\gamma} &\equiv W_{(\alpha_l)}, \end{aligned} \tag{4}$$

we can see that the invariants comprised of the field $W_{(k)}$ and $\widetilde{W}_{(l)}$, $DW_{(m)}$, and $\widetilde{D}\widetilde{W}_{(n)}$ cover all the possibilities of the linearized approximation on the mass shell. Therefore, the general structure of the superinvariants (without using $\gamma^5$ invariance) is

$$\int d^4\Theta d^4x\,[W_{(m)}]^a[\widetilde{W}_{(n)}]^b[DW_{(k)}]^c[\widetilde{D}\widetilde{W}_{(l)}]^d, \tag{5}$$

$$\int d^2\Theta d^4x\,[W_{(m)}]^{a_1}[\widetilde{D}\widetilde{W}_{(l)}]^{b_1}, \tag{6}$$

$$\int d^2\widetilde{\Theta} d^4x\,[\widetilde{W}_{(n)}]^{a_2}[DW_{(k)}]^{b_2}. \tag{7}$$

In Eqs. (5)–(7) $a, b, \ldots b_2$ denote the power of the corresponding fields and the spinor indices are convoluted in every possible way. In the light of the aforementioned problem of the self-dual external field,[1-3] i.e., the field with $\widetilde{W}=0$ (or $W=0$),[1] we examine in Eqs. (5)–(7) the invariants comprised only of the fields $W_{(k)}$ and $DW_{(l)}$, i.e., $b=d=b_1=a_2=0$. These invariants generally do not vanish on the mass shell; however, they have an interesting property-if the external field is a purely gravitational or purely gravity-spin field (spin 3/2), then all these superinvariants vanish (except for the topological invariants $\int d^2\Theta d^4x W^2 \pm \int d^2\tilde\theta d^4x\widetilde{W}^2$). For example, in the purely gravitational case the superfields on the mass shell have a very simple form in the "base 2"

$$W_{\alpha\beta\gamma} = \Theta^\delta C_{\alpha\beta\gamma\delta}\ ;\ D_\alpha W_{\beta\gamma\delta} = C_{\alpha\beta\gamma\delta}, \tag{8}$$

when the external field contains only spin 3/2, we have on the mass shell in the "base 1"

$$W_{\alpha\beta\gamma} = \Phi_{\alpha\beta\gamma}\ ;\ D_\alpha W_{\beta\gamma\delta} = \widetilde{\Theta}^{\dot\xi}\partial_{\dot\xi\alpha}\Phi_{\beta\gamma\delta}. \tag{9}$$

After multiplying the indicated quantities the $D$ components of the corresponding superfields in Eq. (5) and the $F$ components in Eqs. (6) and (7) vanish. Thus, without invoking the $\gamma^5$ invariance, we proved that supergravitation is finite in purely gravitational and purely gravity-spin self-dual fields.

2. Let us examine the constraints imposed on the counterterms by the $\gamma^5$ invariance of supergravitation. We can assume that $\gamma^5$ invariance of the quantum supergravitation, just as $\gamma^5$ invariance of the massless quantum electrodynamics, is a good symmetry, i.e., it holds on the quantum level. In both these theories, the $\gamma^5$ anomaly occurs only as a result of the introduction of interaction with the external axial current, in contrast, for example, to the theory of weak interactions. The $\gamma^5$ invariance of supergravitation in terms of the superfields was formulated by Ogievetsky and Sokatchev.[7] For the values of interest it indicates that the spinor superfield $W_{\alpha\beta\gamma}$ has a chiral

charge $+1$, $\widetilde{W}_{\dot{\alpha}\dot{\beta}\dot{\gamma}}$ has a charge $-1$ and the superfield $D_{\alpha} W_{\beta\gamma\delta}$ and $\widetilde{D}^{\dot{\alpha}} W_{\dot{\beta}\dot{\gamma}\dot{\delta}}$ have a zero chiral charge. Taking also into account that the differentials $d\theta$ have chiral changes of $-1$ and $+1$, respectively, we find that $\gamma^5$ invariance leaves only the following superinvariants in Eqs. (5)–(7)

$$\int d^4\Theta d^4x \, [W_{(m)} \widetilde{W}_{(n)}]^a \, [DW_{(k)}]^c \, [\widetilde{D}\widetilde{W}_{(l)}]^d \tag{5'}$$

$$\int d^2\Theta d^4x \, W^2, \tag{6'}$$

$$\int d^2\widetilde{\Theta} d^4x \, \widetilde{W}^2. \tag{7'}$$

In (6′) and (7′) only the nonvanishing integrals remain from the divergences in the four-dimensional $x$ space.

Let us analyze the invariants comprised only of one kind of fields, for example, $W$ and its derivatives. In (5′) we have at $a=d=0$

$$\int d^4\Theta d^4x \, [DW_{(k)}]^c. \tag{10}$$

Using Eq. (2) we can see that (10) is an integral of the total superdivergence $D\{W_{(l)} [DW_{(m)}]^{c-1}\}$. The single-loop counterterms (6′) and (7′) can be eliminated by renormalizing the topological interaction constant.[2,3] Thus, supersymmetry together with $\gamma^5$ invariance indicates that supergravitation is finite in the self-dual field with $\widetilde{W}=0$ (or $W=0$).

3. We show that the counterterms (5′)–(7′) do not contribute to the quantum equation of motion in the instanton field. For (6′) and (7′) this follows trivially from the fact that they are divergence integrals. We shall prove that in (5′) $a \geqslant 2$ for $d=0$ (or $c=0$), i.e., that neither the local invariants comprised of only the $W$ fields nor the rest comprised of $W$ fields and the first-order $\widetilde{W}$ field contribute to the mass shell. Let us examine in (5′) the cases

$$\int d^4\Theta d^4x \, W_{(m)} \widetilde{W}_{(n)} \, [DW_{(k)}]^c, \tag{11}$$

$$\int d^4\Theta d^4x \, [DW_{(k)}]^c \, \widetilde{D}\widetilde{W}_{(l)}. \tag{12}$$

It can be seen in "base 2" that the $D$ component of the superfields (11) and (12) is equal to zero. Thus, the final answer for the counterterms has a form (5′) with $a \geqslant 2$ at $d=0$ (or $c=0$). This last property indicates that the first variational superfield derivative of the gauge-invariant counterterms (5′) also vanishes in the self-dual field $\widetilde{W}=0$ ( or $W=0$). This property may prove useful in analyzing the quantum equations of motion of the supergravitation.

We note that in terms of the matrix elements of scattering in the $p$ space strong constraints on the $S$ matrix in the supergravitation were obtained in a landmark paper of Grisaru and Pendelton[8] and Grisaru *et al.*[9] Our constraints on the superfield counterterms in the $x$ space apparently are in unique agreement with the result of Ref. 8; however, for quantization purpose in the nontrivial external fields the approach developed here is adequate, whereas the method of standard $S$ matrix is inapplicable.

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*NOTE*: After sending this paper to the editor, I received the preprint of Christensen, Deser, Duff, and Grisaru,[10] in which they also obtained basic constraints on the counterterms in supergravitation, which were mentioned in the first two sections of this paper.

[1)]We used the notations of the landmark paper of Ferrara and Zumino.[6]

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