Interaction of strings in quantum chromodynamics

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The structure of the interaction of hadrons, which are described by the strings with quarks at the ends, is determined in terms of quantum chromodynamics (QCD). The interaction constants are specified by the quantities $\sqrt{\alpha_s}/m_k$, whose values were determined in Preprint ITEP-80, 1978, by Shifman *et al*.

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Let us examine the theory of strong interactions (QCD) with the color gauge group $SU(3)_c$

$$\mathcal{L} = -\frac{1}{4} G^a_{\mu\nu} G^a_{\mu\nu} + \sum_{k=1}^{N} \bar{\psi}_k (i\gamma^\mu D_\mu(A) - m_k)\psi_k, \tag{1}$$

where N is the number of different kinds (flavors) of quarks (color indices of the quarks are omitted). In this theory we can construct gauge-invariant operators (P denotes ordering along a spacelike path and g is the coupling constant in QCD):

$$\Phi(C) \equiv Tr\, P\exp(ig \oint_C A_\mu dx^\mu); \qquad A_\mu = A^a_\mu \frac{\lambda^a}{2},$$

$$\phi^{\Gamma_i}_{k,n}(C_{xy}) \equiv g\bar{\psi}_k(x)\Gamma_i\, P\exp(ig\int_x^y A_\mu dx^\mu)\,\psi_n(y) \equiv g\bar{\psi}_k(x)\Gamma_i\, U(x,y)\psi_n(y),$$

$$Y_{k,n,p}(C_{xyz,v}) \equiv g\epsilon_{abc} U_{aa'}(v,x)\psi_{a'k}(x)U_{bb'}(v,y)\psi_{b'n}(y)U_{cc'}(v,z)\psi_{c'p}(z), \tag{2}$$

$$k, n, p = 1, 2, \ldots N; \qquad a, b, c = 1, 2, 3; \qquad \Gamma_i = 1, \gamma_5, \gamma_\mu, \gamma_\mu\gamma_5, \sigma_{\mu\nu},$$

which correspond to a closed path C (pomeron), an open path C_{xy} with quarks at the end points x and y (meson), and a baryon configuration $C_{xyz,v}$ with quarks at points x, y, and z. Apparently these gauge-invariant operators in a certain approximation describe hadrons in QCD.

Equations for the operators (2) were derived in a number of recently published papers[1–5] [for definiteness, we examine the operators $\phi(C_{xy})$, but similar equations are also valid for the operators $\Phi(C)$ and $Y(C)$]:

$$\frac{\delta^2\phi(C_{xy})}{\delta x_\mu(\sigma)\,\delta x_\mu(\sigma')} = -g^2 x'_\nu(\sigma)x'_\alpha(\sigma')\, g\bar{\psi}(x)\Gamma_i\, P(U(x,y)G_{\mu\nu}(x(\sigma))\, G_{\mu\alpha}(x(\sigma')))\psi(y)$$
$$+\, ig\delta(\sigma-\sigma')\, x'_\nu(\sigma)\, g\bar{\psi}(x)\Gamma_i\, P(U(x,y)D_\mu G_{\mu\nu}(x(\sigma)))\,\psi(y). \tag{3}$$

In an earlier paper,[3] we showed that in the correlation-uncoupling approximation, if only the first term on the right-hand side is taken into account, Eq. (3) becomes an equation for the wave function of a free dual string. The numerical value $\langle(\alpha/\pi)G^a_{\mu\nu} \times G^a_{\mu n}\rangle_0 = 0.013$ (GeV)^4, which was obtained by Vaĭnshteĭn *et al.*[6] by analyzing different sum rules, gives[3] the slope of the Regge trajectories in QCD $\alpha' = 1.1$ GeV^{-2}.

In this paper, we took into account the second term on the right-hand side of Eq. (3), which was omitted by Borisov *et al.*,[3] and determined the specific structure of the interaction of the strings. First, we examine the motion of a quark at the end of the string. We can easily show that, for example, for $\sigma = 0$

$$\frac{\partial\phi^{\Gamma_i}(C_{xy})}{\partial x_\mu(0)} = g\bar{\psi}(x)\overleftarrow{D}_\mu(A)\Gamma_i\, U(x,y)\psi(y), \tag{4}$$

$$\frac{\partial^2\phi^{\Gamma_i}(C_{xy})}{\partial x_\mu(0)\partial x_\mu(0)} = g\bar{\psi}(x)\overleftarrow{D_\mu D_\mu}\Gamma_i\, U(x,y)\psi(y). \tag{5}$$

The Heisenberg equation of motion for the quark field $i\bar{\psi}\overleftarrow{D}_\mu\gamma^\mu = -m\bar{\psi}$ and the relation $D_\mu D_\mu = \hat{D}\hat{D} - (g/2)\sigma_{\nu\alpha}G_{\nu\alpha}$ in the correlation-uncoupling approximation transform Eq. (5) as follows:

$$\frac{\partial^2\phi_{k,n}(C_{xy})}{\partial x_\mu(0)\partial x_\mu(0)} = -m_k^2\phi_{k,n}(C_{xy}), \tag{6}$$

i.e., the motion of the end of the string obeys the Klein-Gordon equation.

The Heisenberg equation of motion for the intensities of the colored field $D_\mu G_{\mu\nu} = \frac{1}{4}g(\bar{\psi}\gamma_\nu\lambda^a\psi)\lambda^a$ and the Firtz relations for direct products of the matrices $\Gamma_i \otimes \gamma_\nu$ and $\lambda^a \otimes \lambda^a$ make it possible to rewrite the second term on the right-hand side of Eq. (3) in the form $[x(\sigma)\equiv z]$:

$$-\frac{i}{32}g\delta(\sigma-\sigma')\,x'_\nu(\sigma)\sum_{r,s=1}^{16} Tr(\Gamma_i\Gamma_r\gamma_\nu\Gamma_s)g\bar{\psi}(x)\Gamma_r U(x,z)\psi(z) \times g\bar{\psi}(z)\Gamma_s U(z,y)\psi(y) \tag{7}$$

(here we dropped the term with the zero-length string). We use the variational-derivative coupling at the end, which occurs as a result of extending the path through the point $x_\mu(0)$, with the partial derivative with respect to the end:

$$\frac{\delta\phi}{\delta x_\mu(0)} = \delta(\sigma)\frac{\partial\phi}{\partial x_\mu(0)},$$

and relation (4) for the operator $\phi^{\gamma^\mu\Gamma_s}(C_{zy})$. Thus, at $\sigma=\sigma'$ Eq. 3 has the form

$$\frac{\delta^2 \phi^{\Gamma_i}_{k,n}(C_{xy})}{\delta x_\mu(\sigma)\,\delta x_\mu(\sigma)} = \frac{1}{(2\pi a')^2}(x'(\sigma))^2 \phi^{\Gamma_i}_{k,n}(C_{xy})$$

$$-\frac{1}{64} x'_\nu(\sigma) \sum_{p=1}^{N} \frac{g}{m_p} \sum_{r,s=1}^{16} \times Tr(\Gamma_i \Gamma_r \gamma_\nu \Gamma_s)$$

$$\times \left[\phi^{\Gamma_r}_{k,p}(C_{xz}) \frac{\overrightarrow{\delta}}{\delta z_\mu} \phi^{\gamma^\mu \Gamma_s}_{p,n}(C_{zy}) - \phi^{\Gamma_r \gamma^\mu}_{k,p}(C_{xz}) \frac{\overleftarrow{\delta}}{\delta z_\mu} \phi^{\Gamma_s}_{p,n}(C_{zy}) \right]. \qquad (8)$$

This equation can be interpreted as a description of the interaction of strings by the ends in the theory of string fields.[7,8] We note that the interaction, which contains the field derivatives, was encountered earlier in the field theory of the interaction of strings with intrinsic spin which corresponds to the Neveu-Schwarts dual model.[9] The structure of the interaction in Eq. (8) is demonstrated in Fig. 1a. An analogous examination of equations for the operators $\Phi(C)$ and $Y(C_{xyz,v})$ gives the interaction of strings shown in Figs. 1b and 1c. It can be seen that the interaction of hadrons in the string approximation of QCD is due to the interaction of valence quarks or, from the viewpoint of another channel, due to the production of a quark-antiquark pair. Note that, according to Eq. (8), the production of colored objects is missing here.

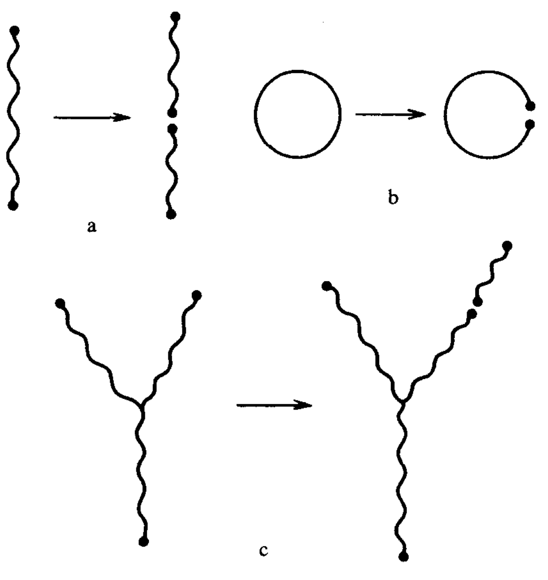

FIG. 1.

Using the equations of motion (6) and (8), we can restore the effective Lagrangian whose variation produces an integrated version of these equations with respect to σ (the other parts of the equations of motion are analogous to the Virasoro algebra generators L_n[10]). This Lagrangian has the following structure:

$$\mathcal{L} = \mathcal{L}_0(\Phi) + \mathcal{L}_0(\phi) + \mathcal{L}_0(Y) + \mathcal{L}_{int}(\phi) + \mathcal{L}_{int}(\phi,\Phi) + \mathcal{L}_{int}(\phi,Y).$$

Let us write explicitly the free Lagrangian for the meson string:

$$\mathcal{L}_0(\phi) = \Sigma \int_0^{\pi} d\sigma \sqrt{-x'(\sigma)^2} \left[\frac{1}{-x'(\sigma)^2} \frac{\delta \phi^+}{\delta x_\mu(\sigma)} \frac{\delta \phi}{\delta x_\mu(\sigma)} - \frac{1}{(2\pi a')^2} \phi^+ \phi \right.$$

$$\left. + (\delta(\sigma) + \delta(\sigma - \pi))(\partial_\mu \phi^+ \partial_\mu \phi - m^2 \phi^+ \phi) \right], \tag{9}$$

where Σ denotes summation over all the indices of the field ϕ (C_{xy}). The meson-meson interaction (Fig. 1a) is described by the Lagrangian:

$$\mathcal{L}_{int}(\phi) = \Sigma \frac{1}{64} \frac{g}{m_p} \int_0^{\pi} d\sigma (\sqrt{-x'(\sigma)^2})^{-1} x_\nu'(\sigma)\, Tr(\Gamma_i \Gamma_r \gamma_\nu \Gamma_s)$$

$$\times \left[\phi^{\Gamma_i}_{k,n}(C_{xy}) \left(\phi^{\Gamma_r}_{k,p}(C_{xz}) \frac{\overrightarrow{\delta}}{\delta z_\mu} \phi^{\gamma^\mu \Gamma_s}_{p,n}(C_{zy}) \right. \right.$$

$$\left. \left. - \phi^{\Gamma_r \gamma^\mu}_{k,p}(C_{xz}) \frac{\overleftarrow{\delta}}{\delta z_\mu} \phi^{\Gamma_s}_{p,n}(C_{zy}) \right) + \text{H.C.} \right]. \tag{10}$$

Note that we used extensively the Heisenberg equations of motion of QCD in the derivation of Eqs. (6) and (8) and hence (9) and (10), i.e., we took into account the nontrivial information on the dynamics of interaction in QCD.

It follows from Eq. (10) that the interaction force of different strings, which is determined by the "flavor" of the created (or annihilated) quarks, is inversely proportional to the mass of these quarks. The quantities g/m_k were estimated phenomenologically[11]

$$\left(\alpha_s \equiv \frac{g^2}{4\pi}\right): \frac{(\alpha_s)^{1/2}}{m_u|_{\mu=0.2\text{ GeV}}} = 0.2\ \text{MeV}^{-1}$$

and

$$\frac{\sqrt{a_s}}{m_u} : \frac{\sqrt{a_s}}{m_d} : \frac{\sqrt{a_s}}{m_s} : \frac{\sqrt{a_s}}{m_c} = 1 : 0.6 : 0.03 : 0.003. \qquad (11)$$

Thus, in terms of QCD in the correlation-uncoupling approximation we constructed an effective string Lagrangian in which all the hadron interaction constants are known.

[1]Y. Nambu, Preprint UT-310, 1978.
[2]Yu. M. Pis'mak, Yad. Fiz. (Sov. J. Nucl. Phys.) (in press).
[3]N.V. Borisov, M.V. Ioffe, and M.I. Eĭdes, Yad. Fiz. **29**, 1421 (1979) [Sov. J. Nucl. Phys. **29**, No. 5 (1979)].
[4]J.L. Gervais and A. Neveu, Preprint LPTENS 78/24, 1978.
[5]E. Corrigan and B. Hasslacher, Prepring LPTENS 78/25, 1978.
[6]A.I. Vaĭnshteĭn, V.I. Zakharov, and M.A. Shifman, Pis'ma Zh. Eksp. Teor. Fiz. **27**, 60 (1978) [JETP Lett. **27**, 55 (1978)].
[7]M. Kaku and K. Kikkawa, Phys. Rev. **D10**, 1110 (1974).
[8]N.V. Borisov, M.V. Ioffe, and M.I. Eĭdes, Yad. Fiz. **21**, 655 (1975) [Sov. J. Nucl. Phys. **21**, 339 (1975)].
[9]N.V. Borisov, M.V. Ioffe, and M.I. Eĭdes, Vestnik LGU **3**, 11 (1976) [Bulletin of the Leningrad State Univ. **3** (1976)].
[10]C. Marshall and P. Ramond, Nucl. Phys. **B85**, 375 (1975).
[11]M.A. Shifman, A.I. Vaĭnshteĭn, and V.I. Zakharov, Preprint ITEP-80, 1978.