Distribution of partons along the transverse momenta in quantum chromodynamics

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It is shown that at sufficiently large Q^2 the semi-inclusive distributions of partons depend on the transverse momentum k through the parameter $\kappa^2 = k^2/Q^2 f$, where f is independent of k.

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The variation of semiinclusive distributions of partons $G_p^{p'}$ along the transverse momentum k and along a part x of the logitudinal momentum of the particle p, which is carried away by the parton p', with increasing momentum transfer Q^2 is determined by the development of the gluon cascade-quark-antiquark pairs with increasing "depth"

$$t(Q^2) = (1/b)\ln\left(1 + (b\alpha_0/2\pi)\ln(Q^2/Q_0^2)\right), \tag{1}$$

where $b = (33 - 2n_f)/6$, n_f is the number of flavors of quarks, Q_0^2 is the momentum transfer for which the initial distributions of partons are defined,[1] and α_0 is the constant of color interaction $\alpha(Q^2)$ at $Q^2 = Q_0^2$.

In the lower order in $\alpha(Q^2)$ the equations for the evolution of the parton cascade can be represented as follows:

$$\frac{\partial}{\partial t} G_p^{p'}(x, \mathbf{k}, t) = \sum_r \hat{L}_r^{p'} G_p^r(x, \mathbf{k}, t), \tag{2}$$

where the operators $\widehat{L}_r^p$ describe the contributions from the processes of emission of gluons by quarks and gluons and the production of quark-antiquark pairs.[1,2]

Multiplying (2) by $(k^2)^l$, integrating over k, and solving the set of equations obtained in this way, we determine the recurrent relations for $K_p^{p'}(x,l,t) = \int dk\,(k^2)^l \times G_p^{p'}(x,k,t)$:

$$K_p^{p'}(x, l, t) = \sum_r \int \frac{dy}{y} \Delta_r^p(x/y, t)(x/y)^{2l} K_p^r(y, l, 0)$$

$$+ \sum_{r,r'} \sum_{n=0}^{l-1} \binom{l}{n}^2 \int_0^t d\tau \frac{dy\, dy'}{y\, y'} Q^{2(l-n)}(\tau)\, \Delta_{r'}^p\left(\frac{x}{y'}, t-\tau\right) \phi_r^{r'}\left(l, n, \frac{y'}{y}\right) K_p^r(y, n, \tau), \tag{3}$$

where $\Delta_r^p(x,t)$ is the Green's function of the equations for $K_p^{p'}(x,0,t)$, $Q^2(\tau)$ is given by the relation (1), and $\phi_p^{p'}$ are expressed in terms of $w_p^{p'}(x)$-probabilities of emission by a p parton of a p' parton which carries away a part of the momentum x,

$$\phi_p^{p'}(l, n, x) = w_p^{p'}(x)\,[x(1-x)]^{l-n} x^{2n}. \tag{4}$$

Using series-recurrent relations (3), we represent each term in the sum on the right-hand side of Eq. (3) as an integral

$$Z(l, m) = \int_0^t d\tau_1 dy_1 dy_1' Q^{2(l-k_1)}(\tau_1)\, \Delta(x/y_1, t-\tau_1)\, \phi(l, k_1, y_1/y_1')$$

$$\int_0^{\tau_1} d\tau_2 dy_2 dy_2' Q^{2k_2}(\tau_2)\, \Delta(y_1'/y_2, \tau_1-\tau_2)\, \phi(k_1, k_2, y_2/y_2') \cdots, \tag{5}$$

$$\int_0^{\tau_{m-1}} d\tau_m dy_m dy_m' Q^{2(k_1-k_2-\dots-k_{m-1})}(\tau_m)\, \Delta(y_{m-1}'/y_m, \tau_{m-1}-\tau_m)$$

$$\times \int dz\, \phi(k_{m-1}, k_1-k_2-\dots-k_{m-1}, y_m/y_m')\, \Delta(y_m'/z, \tau_m)\, K(z, 0, 0).$$

As shown in Ref. 2, the Green's functions $\Delta_p^{p'}(x,t)$ can be represented in the form

$$\Delta_p^{p'}(x, t) = (2\pi i)^{-1} \sum_n \int d\sigma\, x^{-\sigma}\, \overline{\Delta}_p^{p'}(\sigma, n) \exp \lambda_n(\sigma) t, \tag{6}$$

where $\overline{\Delta}_p^{p'}(\sigma,n)$ and $\lambda_n(\sigma)$ are expressed in terms of the Mellin type operators $L_p^{p'}$.[2] Substituting Δ in Eq. (5) as in Eq. (6), we can easily see that each integration over τ_i yields a factor $\alpha(Q^2)Q^{2k_i}$, so that $Z(l,m) = [\dot{\alpha}(Q^2)]^m Q^{2l} U_{l,m}(x,t)/l$

$$U_{l,m}(x,t) = (2\pi i)^{-1} \int d\sigma\, x^{-\sigma} U(l, m, \sigma) \exp\lambda(\sigma) t. \tag{7}$$

Summing the expressions on the right-hand side of Eq. (3), we can see that at sufficiently large Q^2, when $\exp bt \gg \alpha_0 b/2\pi$

$$K_p^{p'}(x, l, t) \approx \alpha(Q^2)\, Q^{2l} \sum_{n=0}^{l-1} a_{n,p}^{p'}(l, x, t)\, [\alpha(Q^2)]^n. \tag{8}$$

It follows from Eq. (8) that at sufficiently large Q^2 in $G_p^{p'}(x,k,t)$ the dependence on k is concentrated in the parameter $\kappa^2 = k^2/Q^2 f_p^{p'}(x,\alpha(Q^2),t)$, where $f_{p_2}^{p'}$ are slowly varying functions of Q^2. In other words, at large Q^2 and fixed x a gauge invariance should be observed in the distribution of the transverse parton momenta.

Since the coefficients $a_{n,p}^{p'}(l,x,t)$ increase rapidly with increasing l, even when $\alpha(Q^2)$ are small, we can determine the main dependence of $K_p^{p'}$ on x and t for sufficiently large l only in two limiting cases: when $x \ll 1$ and when $1 - x = \delta \ll 1$. When $\delta \ll 1$ the σ in Eq. (7) are important (Ref. 2), and we obtain

$$U_{l,m}(x,t) \approx \delta^{8/3\, t + l}\, U_{0,0}(x, t). \tag{9}$$

Hence, $f_p^{p'}(x,\alpha(Q^2),t) \approx \delta C(\alpha(Q^2))$. When $x \ll 1$ the $\sigma \sim 1$ are important (Ref. 2) and the transverse momenta of partons $\langle \mathbf{k}^2 \rangle_p^{p'} = K_p^{p'}(x,l,t)/K_p^{p'}(x,0,t)$ are almost independent of x: only the transverse momenta of quarks increase weakly $[\sim(\ln(1/x))^{1/2}]$ with decreasing x, because of "shoaling" of the quark-antiquark sea. Thus, when x's are small $\kappa^2 = \mathbf{k}^2/Q^2 C[\alpha(Q^2)]$ and comparing it with $\kappa^2 = \mathbf{k}^2/Q^2 C'[\alpha(Q^2)]$ as $x \to 1$, we see that the distribution of partons contracts greatly with increasing x and becomes narrower by a factor of $1/\delta$ in the quasi-elastic region than in the Reggeon region.

Let us evaluate the rms transverse momentum of partons in the nucleon in the model in which all the quarks, antiquarks, and gluons are produced from the field clouds of three valence quarks which exist at $Q_0^2 \approx \mu^2$, where μ is the average transverse momentum of the valence quarks at this momentum transfer, i.e., in fact, the reciprocal radius of the "infrared jail."[3,4] Apparently estimate is valid only if the perturbation theory and hence Eq. (2) can be used for those Q^2 for which only three valence quarks exist. We assume that this extrapolation to some extent is permissible.

We assume that the distribution function of the valence quarks $G_h^v(x,k,0)$, which has a peak at $x = 1/3$, is normalized in such a way that

$$\int G_h^v(x, \mathbf{k}, 0)\, dx\, d\mathbf{k} = 3; \qquad \int x G_h^v(x, \mathbf{k}, 0)\, dx\, d\mathbf{k} = 1,$$
$$\int \mathbf{k}^2 G_h^v(x, \mathbf{k}, 0)\, dx\, d\mathbf{k} = 3\mu^2. \tag{10}$$

Thus, using relations (3) we can easily show that in the limiting cases $1 - x = \delta \ll 1$ and $x \ll 1$ the transverse rms momenta of partons $\langle \mathbf{k}^2 \rangle_h^p$ are independent of the choice of $G_h^v(x,k,0)$. At $\delta \ll 1$, we obtain for the quarks and gluons, respectively,

$$\langle \mathbf{k}^2 \rangle_h^q = \mu^2 + 4\alpha(Q^2)\, Q^2\, \delta/3\pi\,,$$
$$\langle \mathbf{k}^2 \rangle_h^g = \mu^2 + 3\alpha(Q^2)\, Q^2 \delta/\pi\,. \tag{11}$$

Assuming that $\alpha(Q^2) = 4/9\ \ln(Q^2/\mu^2)$ at $Q^2 = 4$ GeV^2, $\mu = 0.3$ GeV, and $\delta = 0.1$, we see that the transverse rms momenta of partons are $\sim\mu^2$. At $x \ll 1$ $\langle \mathbf{k}^2 \rangle_h^p \approx \alpha(Q^2) f(x,t)$, where f depends weakly on x and t. At $x = 0.1$ and $Q^2 = 4$ GeV^2, we obtain $\langle \mathbf{k}^2 \rangle_h^q \approx 0.6$ GeV^2 for the transverse rms momentum of the quarks, which is close to that reported by Field.[5] Under the same conditions $\langle \mathbf{k}^2 \rangle_h^g \sim 1$ GeV^2.

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