Formation of fast-electron "tails" in a strong Langmuir turbulence

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Numerical modeling of 1-*D* Langmuir turbulence is carried out. Self-consistent spectra of waves and particles in the turbulence are found and the integral characteristics of the turbulent state are calculated.

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In contrast with the hydrodynamic turbulence of an incompressible liquid the Langmuir turbulence lends itself to 1-*D* modeling. This circumstance, first noted by Sagdeev,[1] is associated with preservation in the 1-*D* case of the basic property of strong turbulence of the plasma waves—a tendency toward development of a modula-

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tion instability and localization of the wave energy in cavities. In this paper we report the results of numerical modeling of the 1-*D* Langmuir turbulence which takes into account the self-consistent relationship between waves and particles.

The system of equations considered here was first obtained by Zakharov[2] and it consists of an equation for the complex amplitude of the electric field of the plasma waves $E(t,x)$ ($E_p = \frac{1}{2}E(t,x)e^{-i\omega_p t} + k$ and a density equation $\delta n(t,x)$ for slow quasi-neutral plasma movements that occur under the effect of high-frequency pressure:

$$2i\,(E_t + \hat{\Gamma} E) + E_{xx} = \delta n E - \langle \delta n E \rangle ,$$
$$\delta n_{tt} = \hat{\gamma}\delta n_t - \delta n_{xx} = |E|^2_{xx} \tag{1}$$

Introduction of the last term into the right-hand side of an equation for the electric field corresponds to the presence of longwave pumping maintained at a constant level

$$\langle E \rangle = \frac{1}{L}\int_0^L dx\, E(t\ x) = E_0, \quad E_{0t} = 0 \tag{2}$$

(here we consider the special case for which the pumping frequency is considered exactly equal to the plasma frequency).

It was shown [3] that such treatment of pumping facilitates the modeling of turbulence that is initiated by an electromagnetic wave propagating in a plasma, whence E_0 is the amplitude of a longwave (in the plasma turbulence scale) electromagnetic wave.

In addition to considerations cited above,[2] Eq. (1) takes into account the absorption of the plasma and acoustic waves by particles. Here, $\hat{\Gamma}E$, $\hat{\gamma}\delta n$ are the integral convolution operators that determine the corresponding absorption, for example:

$$\hat{\Gamma}E = \int_0^L \Gamma(t\ x - x\ )\,E(t,\ x\ )\,dx\ , \qquad \Gamma(t,x) = \frac{1}{2\pi}\int_{-\infty}^{\infty}\Gamma_k(t)\,e^{ikx}\,dk.$$

The attenuation decrement of the plasma waves is found allowing for a quasi-linear deformation of the resonant electron distribution function:

$$\Gamma_k = \frac{9}{2}\sqrt{\frac{\pi}{2}}\left(\frac{M}{m}\right)^{3/2}\nu^2\,\frac{\partial f}{\partial \nu}\ , \quad k\nu = 1,$$
$$\frac{\partial f}{\partial t} = \frac{4m}{9M}\,\frac{\partial}{\partial \nu}\left[\frac{|E_k|^2}{\nu}\,\frac{\partial f}{\partial \nu}\right], \quad |E_k|^2 = \frac{1}{L}\left|\int_0^L dx\, e^{-ikx} E(t,x)\right|^2 \tag{3}$$

For γ_k's, we use the normal linear attenuation decrement of the acoustic waves in a non-isothermal ($T_e > T_i$) plasma: $\gamma_k = [(\pi/8)\,(m/M)]^{1/2}\,K$. The system of equations (1)–(3) is expressed in terms of dimensionless variables and it makes use of the following measuring scales for t, x, v, δn, E and f:

$$\bar{t} = \frac{3M}{m}\,\frac{1}{\omega_p}\ , \quad \bar{x} = 3\sqrt{\frac{M}{m}}\,r_D, \quad \bar{v} = 3\sqrt{\frac{M}{m}}\,v_T, \tag{4}$$

$$\delta n = n_0\,\frac{m}{3M}\ , \quad \bar{E} = \left(\frac{16\pi}{3}\,n_0\ T\,\frac{m}{M}\right)^{1/2}, \quad \bar{f} = \frac{n_0}{\sqrt{2\pi}\,v_T}\ .$$

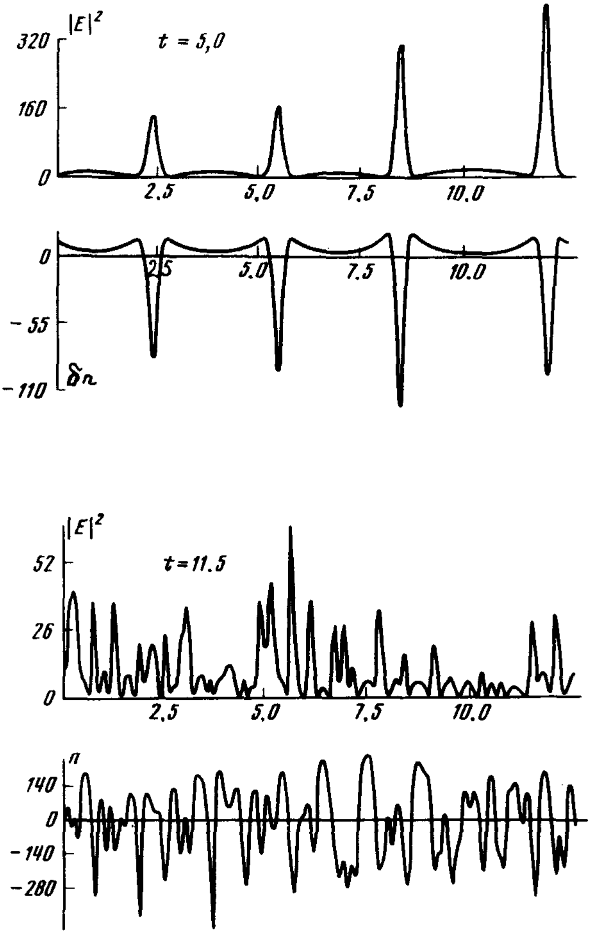

FIG. 1. Spatial distributions of $|E|^2$ and δn in the Langmuir turbulence at $E_0 = 2$.

Equations (1)–(3) were numerically integrated over a finite interval $L = 4\pi$, with periodic boundary conditions for all the unknowns up to time $t \approx 10$–30 ($\sim(6\times10^4$–$2\times10^5)\omega_p^{-1}$). The initial conditions used in the turbulence modeling were the random noise distributions of the electric field and density variations (see for example Ref. 4). To integrate Eq. (1) we used the numerical Fourier method with the full number of harmonics $N = 512$. The integration interval for Eq. (3) was limited by $v_{\max} = 1/k_{\min} = 2$. The parameter of the problem was the dimensionless pumping amplitude E_0.

Figure 1 shows a typical view of the dynamics of the spatial field and density distribution obtained at $E_0 = 2$. In the initial stage, pumping produces an almost periodic soliton lattice with a wavelength corresponding to the maximum of modulation instability increment (Fig. 1a). Subsequently, energy transfer among the interacting solitons, the occurrence of "high" solitons and the resonant absorption of the plasma waves bounded in them by the particles leads to the destruction of such lattice. With time, the picture is agitated further (see transition to Fig. 1b). A large number of randomly distributed solitons of different amplitude occur; in conjunction with these intense shortwave modulation of the δn curve takes place which is dependent on the accumulation of acoustic waves emitted by the collapsed solitons.

As a result of absorption of the plasma waves the resonant electrons sprout a "tail," absorption occurs at lower wave numbers, and the soliton amplitudes in a full-turbulence mode are considerably smaller than those in Ref. 4 where the turbulence was modeled for the case of an invariant resonant particle distribution function. The presence of intense oscillations and the sprouting of the resonant electron "tail" in the direction of small k leads to the occurrence of a new channel of the shortwave transformation of the Langmuir waves—direct transfer from the source region into the absorption region due to conversion on the accumulated sound; the characteristic increment of this process is:

$$\gamma_{\rm conv} = \frac{1}{9} \sum_k \frac{\Gamma_k}{k^4 r_D^4} \frac{|\delta n_k|^2}{n_0^2} \tag{5}$$

A comparison of the Langmuir and acoustic spectra (Fig. 2) shows the conversion in the full-turbulence mode constitutes the fundamental mechanism of shortwave transfer of the plasma waves. The Langmuir solitons occur in the region of wave numbers $k = k_0\ [(m/M)(W/n_0T)]^{1/2}(1/r_D)$ which in Fig. 2 corresponds to approximately the 10th spectral harmonic. Concurrently, a maximum of the acoustic and Langmuir spectra (conversion region) takes place at $k = 15$–20, i.e., the inertial interval in the problem under consideration actually disappears due to "tail" sprouting. In this case, in addition to Eq. (5) the condition $\Gamma(\bar{k}) = \gamma_{\rm mod}$ is satisfied which, when

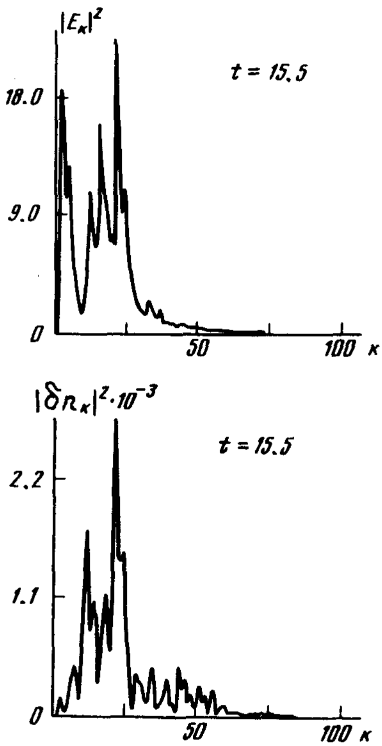

FIG. 2. Plasma $|E_k|^2$ and acoustic $|\delta n_k|^2$ wave spectra at $E_0 = 2$.

combined with Eq. (5) yields

$$\sum_k \frac{|\delta n_k|^2}{n_0^2} \approx (3\bar{k}^2 r_D^2)^2 .$$

The above relationship was satisfied to 10–20% accuracy (for numerical modeling). In a mode without the inertial interval, $\bar{k} \sim k_0 \sim W^{1/4}$ and, therefore

$$\sum_k \frac{|\delta n_k|^2}{n_0^2} \sim W .$$

As a result of growth of the attenuation decrement of the plasma waves Γ_k with the wave number k all the solitons in the absorption region are approximately of the same scale $\bar{k}$. In this case the Langmuir spectrum at $k > \bar{k}$ is exponential

$$|E_k|^2 \sim \exp(-k/\bar{k}).$$

In the steady state, we get $(|E_k|^2/v)(\partial f/\partial v) = \mathrm{const}$ in the quasi-linear diffusion equation from a condition for the invariance of the resonant particle flow, i.e.,

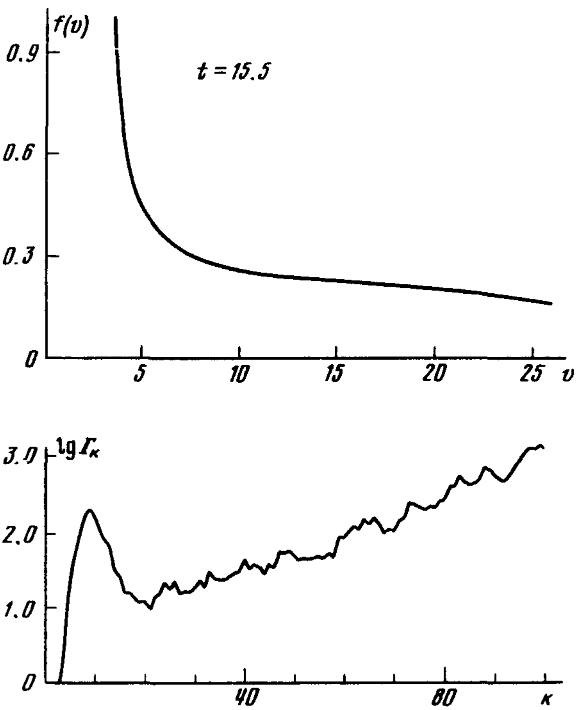

FIG. 3. Plasma wave attenuation decrement Γ_k and resonant electron distribution functions $f(v)$ at $E_0 = 2$. Distribution function corresponds to the presence of a constant flow of resonant electrons into a region of high velocities, thus causing the velocity distribution in the region $v > v_{cr}$ to be stationary although the upper "tail" boundary v_{max} increases with time.

$$\Gamma_k \sim \frac{\partial f}{\partial v} \sim \frac{1}{|E_k|^2} \sim \exp \{ k/\bar{k} \} . \qquad (6)$$

Precisely, this kind of dependence of the attenuation decrement on the wave number was obtained in the course of numerical modeling: at $k > 25$ $\Gamma_k \approx \exp(+0.035k)$ (see Fig. 3a); the corresponding resonance electron distribution function is as follows (for the sake of convenience we shall express it in dimensional variables)

$$f(v) \approx \exp\left\{ 0.1 \sqrt{\frac{M}{m}} \frac{v_T}{v} \right\} - 1 . \qquad (7)$$

The "tail" forms at velocities $v > v_{cr}$ (Fig. 3b). The limiting velocity determines the number of particles in the "tail"

$$n_{\text{tail}} = n_o \sqrt{\frac{2}{\pi}} \int_{v_{cr}}^{\infty} e^{-\frac{v^2}{2v_T^2}} \frac{dv}{v_T} ;$$

this velocity is practically independent of time and is a single-valued function of the pumping amplitude E_0.

The balance between the pumping of energy into a turbulence due to modulation instability and its absorption by the "tail" electrons leads to establishment of a quasi-steady-state level of the plasma oscillations $W = \Sigma_k |E_k|^2/8\pi$. The rate of energy pumping into the turbulence is characterized by the effective frequency of collisions ν_{eff} which is determined from the balance condition

$$\nu_{\text{eff}} = \sum_k \Gamma_k |E_k|^2 / E_o^2 .$$

In the dynamically-stationary state of turbulence energy absorbed from the pumping is expended to accelerate the "tail" particles. The energy of these grows linearly with time. The slow decline of the distribution function at large values of $v(f(v) \sim 1/v)$ leads to a large energy content in the "tail." Thus, at $E_0 = 2$ and $t = 14$ the number of particles $n(v)$ with $v \gg 3v_T$ is $1.3 \times 10^{-2} n_0$ and their energy $E(v) = 1.3\ n_0 T$. Actually, at these times the contibution of the "tail" to dispersion of the Langmuir waves is considerable.

The integral characteristics of the turbulent state that occurs at various pumping amplitudes E_0 are shown in a table above. The stationary level W and, consequently, ν_{eff} are significantly smaller than in the earlier formulation[4] with a fixed resonant electron distribution function.

In the full conversion mode, collapsing supersonic cavities occur only in numbers that are necessary to sustain the stationary level of sound in the presence of attenuation. The energy in these cavities is determined from the balance condition (see

Ref. 5).

$$\gamma_{\text{mod}} \bar{k}^2 r_D^2 W_{\text{cav}} \approx \gamma(\bar{k})\, n_o T \sum_k \frac{|\delta n_k|^2}{n_o^2} . \qquad (8)$$

According to statements in Ref. 5, $\nu_{\text{eff}} \sim W_{\text{cav}}$. Thus, Eq. (8) yields $\nu_{\text{eff}} \sim W^{1/4}$ and the balance equation for the Langmuir oscillations yields $W \sim (E_0^2)^{4/5}$. These analytical

TABLE I.

E_0	$W/n_0 T$	ν_{eff}/ω_p	v_{cr}/v_T	n_{tail}/n_0	γ_{conv}/ω_p
1.0	0.0020	0.012	2.8	8×10^{-3}	1.65×10^{-3}
2.0	0.0055	0.016	2.3	20×10^{-3}	5.3×10^{-3}
4.0	0.0160	0.035	1.7	90×10^{-3}	1.37×10^{-2}

dependences of ν_{eff} and W on E_0^2 satisfactorily agree with expressions obtained in the numerical experiment (see Table I).

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