Magnetic solitons propagating along the anisotropy axis

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It is shown that for magnetic solitons, which propagate along the anisotropy axis (the calculation of) the magnetic-dipole interaction is not simply reduced to a renormalization of the anisotropy constant, but depending on the assumptions about the state of the magnetic medium (background) may significantly change the structure of the solitons. It was found that under certain conditions an isolated wave with exponential asymptotics is formed.

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1. The magnetic solitons, which propagate along the anisotropy axis, were examined for the first time by Akheizer and Borovik.[1] Later papers[2-5] maintain that the (calculations of) demagnetization fields (magnetic dipole interaction) lead to a simple renormalization effect of the energy constant of the uniaxial anisotropy K_1. In this paper we show that, in addition to renormalization of the anisotropy constant, the magnetic-dipole interaction produces a constant internal magnetic field directed along the anisotropy axis. As a result, we have two types of solitons, which are characterized by a different dependence of the limiting velocity u_{max} on the parameter of the magnetic medium $\epsilon = 2\pi M_s^2/K_1$, where M_s is saturation magnetization. The first type of soliton, which exists at $\epsilon < 1$, propagates against the background of a uniform precession of the magnetic moment around the anisotropy axis with the frequency ϵ; as $u \to u^{I}_{max} = 2\sqrt{1-\epsilon}$ the solution goes over to the spin waves of a vanishingly small amplitude. The second type of soliton exists for all ϵ. At $\epsilon \leqslant 2$ and $u \to u^{II}_{max} = 2$ it also goes over to the spin waves of a vanishingly small amplitude; however, at $\epsilon \geqslant 2$ $u \to u^{II}_{max} = 2$ a transition occurs to the spin waves of the finite amplitude. This transition occurs via a soliton of finite amplitude with power-law asymptotics.

2. Let us examine the Landau-Lifshitz equations

$$\frac{\partial \mathbf{m}}{\partial t} = \left[\mathbf{m} \times \frac{\delta W}{\delta \mathbf{m}}\right], \tag{1}$$

where W for a ferromagnet with an anisotropy such as "easy axis of magnetization" has the form (we assume that the solutions depend on the space variable z, which coincides with the anisotropy axis)

$$W = \left(\frac{\partial \theta}{\partial z}\right)^2 + \sin^2\theta\left(\frac{\partial \phi}{\partial z}\right)^2 + \sin^2\theta + \frac{{h^{(i)}}^2}{\epsilon}. \tag{2}$$

Here θ and ϕ are the polar and azimuthal angles of the magnetic moment vector and the polar axis coincides with the anisotropy axis. The demagnetizing field $h^{(i)}$ is determined by magnetostatics equations whose solution in the geometry under consideration is as follows:

$$h_x^{(i)} = h_y^{(i)} = 0; \qquad h_z^{(i)} = \epsilon(1 - \cos\theta). \tag{3}$$

Equation (3) takes into account that the demagnetizing field vanishes in the region of the homogeneous state $\theta = 0$. Substituting Eq. (3) in Eq. (2) we can see that the magnetic-dipole interaction produces a double effect: the term $\epsilon \cos^2\theta$ renormalizes the anisotropy constant (which is equal to unity in the selected notations); moreover, there appears a term $2\epsilon \cos\theta$, which plays the role of a constant effective field of magnitude ϵ, which is directed along the anisotropy axis. We emphasize once more that the external magnetic field is missing. We obtain from Eqs. (1)–(3) the following equations

$$-\sin\theta \frac{\partial\phi}{\partial t} = \frac{\partial^2\theta}{\partial z^2} - \left[1 + \left(\frac{\partial\phi}{\partial z}\right)^2\right] \sin\theta\cos\theta - \epsilon(1 - \cos\theta)\sin\theta,$$

$$\sin\theta \frac{\partial\theta}{\partial t} = \frac{\partial}{\partial z}\left(\sin^2\theta \frac{\partial\phi}{\partial z}\right). \tag{4}$$

Equations (4) for solutions of the type[1)]

$$\theta(z, t) = \theta(\zeta); \qquad \phi(z, t) = \epsilon t + \Phi(\zeta); \qquad \zeta = z - ut \tag{5}$$

which correspond to the stationary profile waves propagating against the background of a uniform precession of the magnetic moment around the anisotropy axis with a frequency ϵ, reduce to the first integral

$$\left(\frac{d\theta}{d\zeta}\right)^2 + u^2 \frac{1 - \cos\theta}{1 + \cos\theta} - (1 - \epsilon)\sin^2\theta = 0. \tag{6}$$

A simple analysis of the solutions, which correspond to the first integral (6), shows that the velocity of the solitons is bounded above by the quantity $u^{\mathrm{I}}_{\max} = 2\sqrt{1-\epsilon}$, which determines the interface of the soliton states and the spin waves with a vanishingly small amplitude. The limiting velocity of solitons vanishes when the characteristic anisotropy field is equal to the saturation field ($\epsilon = 1$). The existence domain of the soliton states is limited by the condition $\epsilon < 1$. The corresponding solutions have the form

$$\cos\theta = 1 - \frac{2 a u e^{-\sqrt{a}\zeta}}{\sqrt{1-\epsilon}\{(u e^{-\sqrt{a}\zeta} + 2\sqrt{1-\epsilon})^2 - a\}} \tag{7}$$

$$a = 4(1 - \epsilon) - u^2.$$

3. Let us examine another type of solution

$$\theta(z, t) = \theta(\zeta); \qquad \phi(z, t) = \phi(\zeta), \tag{8}$$

which corresponds to the stationary profile waves propagating in an unperturbed medium. Equations (4) lead to the first integral

$$\left(\frac{d\theta}{d\zeta}\right)^2 + u^2\frac{1-\cos\theta}{1+\cos\theta} - \sin^2\theta - \epsilon(1-\cos\theta)^2 = 0. \tag{9}$$

In this case the analysis of the solutions shows that the velocity of the solitons is bounded from above by the quantity

$$u^{II}_{max} = 2, \tag{10}$$

which is independent of the characteristic parameter of the magnetic medium ϵ, which determines the contribution of the magnetic-dipole interaction. The limiting velocity (8) determines the interface of the soliton states and the spin waves. However, the solution depends on the magnitude of ϵ. The plane of the parameters (ϵ,u) has four regions shown in Fig. 1.

I. $\epsilon<2$ and $u<2$. The soliton solutions have the form

$$\cos\theta = 1 - \frac{2(4-u^2)\sqrt{\epsilon^2-u^2(\epsilon-1)}\,e^{-\sqrt{4-u^2}\zeta}}{[\sqrt{\epsilon^2-u^2(\epsilon-1)}\,e^{-\sqrt{4-u^2}\zeta}+(2-\epsilon)]^2+(4-u^2)(\epsilon-1)}. \tag{11}$$

As $u\rightarrow 2$ the amplitude approaches zero. The line $u=2$, $\epsilon\leqslant 2$ is the interface of the soliton states (1) and the region of the spin waves of the vanishingly small amplitude (IV).

II. $\epsilon\geqslant 2$, $u<2$. There are soliton solutions such as (11). However, as $u\rightarrow 2$ the amplitude of the soliton remains finite but the characteristic length of the soliton $\zeta_0 = 1/\sqrt{4-u^2}\rightarrow\infty$. At the line $u=2$, $\epsilon\geqslant 2$ which divides the region of the soliton states (II) and the spin waves of the finite amplitude (III) the solution is a soliton with power-law asymptotics

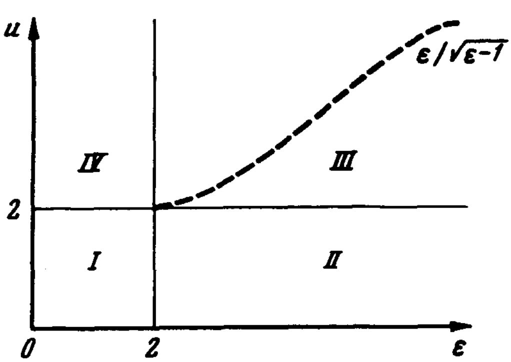

FIG. 1.

$$\cos\theta = 1 - \frac{2}{(\epsilon-2)\zeta^2 + \dfrac{\epsilon-1}{\epsilon-2}} \quad . \tag{12}$$

III. $\epsilon \geqslant 2$, $2 < u < \epsilon/\sqrt{\epsilon-1}$. The region of spin waves of finite amplitude. The vector of the magnetic moment, which precesses around the anisotropy axis, nutates along the polar angle θ:

$$\cos\theta = 1 - \frac{4-u^2}{(2-\epsilon) + \sqrt{\epsilon^2 - u^2(\epsilon-1)}\,\cos[\zeta\sqrt{u^2-4}]} \quad . \tag{13}$$

As $u \to 2$ the solution of Eq. (13) continuously goes over to the soliton with power asymptotics and as $u \to \epsilon/\sqrt{\epsilon-1}$ the nutation amplitude approaches zero and the rotation of the vector of the magnetic moment remains uniform around the anisotropy axis with a constant polar angle $\theta_0 = \text{arc}\cos x[1/(\epsilon-1)]$.

4. The formation of an isolated wave with power asymptotics at $\epsilon \geqslant 2$ and $u \to 2$ is attributable to the degeneracy of the singular saddle point when the limiting velocity is attained. Since the transition from the soliton states to the spin waves occurs at finite amplitudes of solutions, these soliton states cannot be predicted by means of continuing the spectrum of the spin waves to the region of complex wave numbers, as developed by Bar'yakhtar *et al.*[6] According to Eq. (12), the maximum amplitude and the region of localization of the power soliton are determined by the characteristic parameter of the magnetic medium ϵ.

Thus, the magnetic-dipole interaction, which can have a strong effect on the structure of the solitons propagating along the easy axis of anisotropy, is determined by the character of assumptions regarding the magnetic medium (background).

[1)] This type of solitons were examined in Ref. 5 in the presence of a constant external magnetic field but without taking into account the magnetic-dipole interaction.

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