Critical fluctuations in liquid He^3: Stabilization of the Anderson-Morel phase

A. I. Sokolov

V. I. Ul'yanov (Lenin) Leningrad Institute of Electrical Engineering

(Submitted 23 March 1979)

Pis'ma Zh. Eksp. Teor. Fiz. **29**, No. 10, 618–622 (20 May 1979)

The equations for the renormalized group, which describe the superfluid phase transitions in liquid He^3, are derived and solved (on the computer). It is shown that the interaction of the critical fluctuations of the order parameter increases the region corresponding to phase A in the phase diagram of He^3.

PACS numbers: 67.50. – b, 64.70.Ja

It is known that liquid He^3 at temperatures below 2.6 mK can exist in two superfluid modifications. One of them, which is characterized by an anisotropic gap in the spectrum of elementary excitations (phase A), was studied theoretically for the first time by Anderson and Morel.[1] The other phase, which has a spectrum with an isotropic gap (phase B), was described by Balian and Werthamer.[2] They also showed that in the weak coupling approximation, i.e., in terms of the BCD theory, only phase B is thermodynamically stable below the superfluid phase transition point. To explain the existence of phase A at pressures above 21 atm, Anderson and Brinkman[3] went beyond the weak-coupling approximation and took into account the renormalization of the peak due to the spin fluctuation exchange. It was found that the paramagnon exchange indeed stabilizes the A-phase and allowance for the sixth-order invariants in the free-energy expansion makes it possible to understand qualitatively the structure of the phase diagram of liquid He^3 as a whole.[4]

The Anderson-Brinkman theory as applied to the superfluid phase transitions in He^3 is analogous to the Landau theory in the sense that it ignores the critical fluctuations of the order parameter. These fluctuations can be legitimately ignored if the Ginzburg-Levanyuk parameter for the system under consideration is small, which is usually attributed to the relatively weak interaction responsible for the phase transition. In the case of liquid He^3, however, the effective interaction apparently is sufficiently large, since the BCS theory is not applicable here and hence the Ginzburg-Levanyuk parameter for the superfluid phase transitions must be relatively large. The importance of the critical fluctuations of the order parameter is indicated by the results of recent experiments on absorption of the zero sound in a normal He^3 in the neighborhood of the transition of the superconducting state.[5] Therefore, a question is raised concerning the role of the critical fluctuations in determining the super-fluid phase transition to He^3 and in forming its phase diagram. The purpose of this communication is to determine this function.

We use the following fluctuation Hamiltonian, which describes the superfluid transition in liquid He^3:

$$H = \frac{1}{2} \int dx \Big[(\kappa_o^2 + \nabla^2) \phi_{ij} \phi_{ij}^* + f(\nabla_i \phi_{ki})(\nabla_j \phi_{kj})$$

$$+ \frac{1}{2} (\beta_1^{(o)} \phi_{ij} \phi_{ij} \phi_{kl}^* \phi_{kl}^* + \beta_2^{(o)} \phi_{ij} \phi_{kl} \phi_{ij}^* \phi_{kl}^* + \beta_3^{(o)} \phi_{ij} \phi_{kj} \phi_{kl}^* \phi_{il}^*$$

$$+ \beta_4^{(o)} \phi_{ij} \phi_{kl} \phi_{kj}^* \phi_{il}^* + \beta_5^{(o)} \phi_{ij} \phi_{il} \phi_{kj}^* \phi_{kl}^*) \Big]. \quad (1)$$

Here $\phi_{ij}(x)$ is a complex tensor field of the fluctuations of the order parameter, where the first index of ϕ_{ij} is the spin index and the second is the orbital index. The parameter f determines the anisotropy of the fluctuation spectrum and $\beta_\alpha^{(0)}$ ($\alpha = 1,2,3,4,5$) play the part of the priming coupling constants. The spin-orbit terms in Eq. (1) were dropped because of their smallness.

To determine the critical behavior of the system, we must have the equations for the renormalization group (RG), which control the evolution of the physical charges β_α at $T \to T_c$. Since the structure of the ordered phase is not determined by the magnitudes of β_α but rather by their ratios, it is sufficient to analyze only the equations for these ratios. As the sought-for functions it is convenient to select linear combinations of the ratios β_α/β_1 ($\alpha = 2,3,4,5$), which appear in the conditions of thermodynamic stability of the phases A and B[6]:

$$v = \frac{\beta_2}{\beta_1}, \quad x = \frac{\beta_4 + \beta_5}{\beta_1} \quad \gamma = \frac{\beta_3}{\beta_1}, \quad z = \frac{\beta_5 - \beta_3}{2\beta_1}. \quad (2)$$

The full set of equations for the RG will be derived and analyzed in another paper. Here we would only like to point out that the theory can be radically simplified if the anisotropy of the fluctuation spectrum is ignored. At the same time, allowance for the anisotropic term in the Hamiltonian (1), as the estimates show, should not substantially influence the results. Therefore, we use the equations for the RG obtained at $f = 0$. In the lowest (parquet) approximation they have the form:

$$\frac{dv}{dt} = -\beta_1 [7v^2 + 3x^2 + 3\gamma^2 + 8z^2 + 10\,vx - 6v\gamma - 16\,vz - 2x\gamma - 8xz$$
$$+ 8\gamma z - 5v + 4 + 4\,v\gamma(\gamma + 2z)],$$

$$\frac{dx}{dt} = -\beta_1 [4x^2 + 7\gamma^2 + 8z^2 - 8x\gamma - 8xz + 8\gamma z - 5x + 8\gamma - 4x\gamma(\gamma + 2z)],$$

$$\frac{d\gamma}{dt} = -\beta_1 [-15\gamma^2 - 24\gamma z + 8x\gamma + 4x - 9\gamma - 4\gamma^2(\gamma + 2z)],$$

$$\frac{dz}{dt} = -\beta_1 z [8x - 14\gamma - 22z - 13 - 4\gamma(\gamma + 2z)], \qquad t = c/\kappa. \quad (3)$$

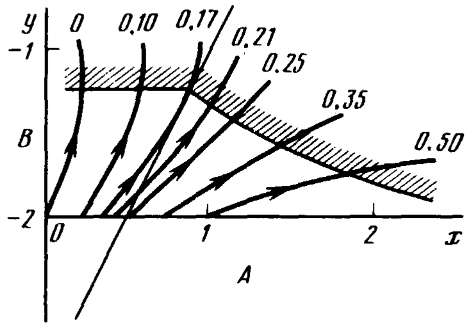

FIG. 1. Projections on the x-y plane of the three-dimensional phase trajectories of the RG equations (3). The straight line fixes the limits of the stability zones of the phases A and B. The crosshatched area represents the boundary of the stability region of the Hamiltonian (1). The numbers on the curves are equal to the corresponding values of the parameter δ.

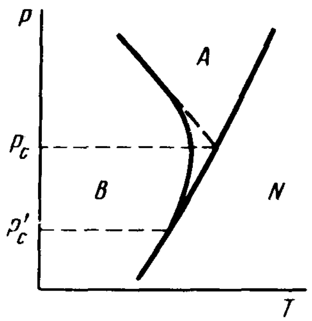

FIG. 2. The phase diagram of He^3 taking into account the critical fluctuations. The letter N denotes the existence domain of the normal Fermi liquid. The heavy broken line represents the boundary of phase A in the Anderson-Brinkman-Serene theory; $\delta(P_c) = 0.25$ and $\delta(P_c') = 0.17$.

Here κ is the inverse correlation radius and the constant $c > 0$. As can be seen, the last three equations form a closed system, which has eight fixed points along which there are stable nodes. However, Eq. (3) on the whole does not have stable solutions, since $|v| \to \infty$ as $\kappa \to 0$, regardless of the evolution of x, y, and z. Here the sign of dv/dt is such that the fourth order in (1) loses definiteness for any initial values of β_α. Thus, the superfluid phase transition in He^3 in principle must be a first-order transition.

Let us assume that the initial values of β_α coincide with those which the Anderson-Brinkman[3] paramagnon theory gives:

$$\beta_2^{(0)} = -(2+\delta)\beta_1^{(0)}, \quad \beta_3^{(0)} = -2\beta_1^{(0)}, \quad \beta_4^{(0)} = -(2-\delta)\beta_1^{(0)},$$

$$\beta_5^{(0)} = (2+\delta)\beta_1^{(0)}, \quad \beta_1^{(0)} < 0. \tag{4}$$

How does the ratio of the constants β_α vary under the action of the critical fluctuations and how does the value δ influence the structure of the low-temperature phase? The answers to these questions are given in Fig. 1 which shows the x-y projections of the three-dimensional phase trajectories $[x(t), y(t), \text{and } z(t)]$ of the equations for the RG, which begin on the straight line parametrized by Eqs. (4). It also shows the range of the parameters x and y within which the phases A and B, respectively, are stable and the boundary of the stability region of the Hamiltonian (1). It can be seen in Fig. 1 that at $\delta < 0.17$ and $\delta > 0.25$ the phase trajectories of the equations for RG cross the boundary of the stability region of the Hamiltonian in the same zones in which they originate. Apparently the fluctuations of the order parameter do not change the ratios of the free energies of the phases A and B.

Such a variation occurs, however, at $0.17 < \delta < 0.25$. In this case the phase trajectories, which begin in the stability region of phase B, go to the stability region of phase A, which indicates that phase A is stabilized by the critical fluctuations. If δ is in the range of 0.17 to 0.25, first a transition to phase A occurs in the system and then as the temperature and the fluctuations decrease this phase is replaced by phase B by means of a first-order phase transition.

As is known, the parameter of the paramagnon coupling δ increases with pressure P. It follows from this that the P-T diagram of the liquid He^3 should have an additional stability region of phase A, which is adjacent to the region described by the Anderson-Brinkman-Serene theory[4] and which has the shape of a beak, as shown in Fig. 2. The characteristic width of this "beak" apparently does not exceed the width of the critical region. It is curious that the appearance of a beak in the phase diagram of He^3 is not connected with anisotropy of the critical-fluctuation spectrum, as is the case for cubic and tetragonal crystals with dipole forces.[7,8]

I sincerely thank E.B. Sonin for stimulating discussions, for discussion of the results, and also for collaboration during the initial stage of this work.

1P.W. Anderson and P. Morel, Phys. Rev. Lett. **5**, 136 (1960).
2R. Balian and N.R. Werthamer, Phys. Rev. **131**, 1553 (1963).
3P.W. Anderson and W.F. Brinkman, Phys. Rev. Lett. **30**, 1108 (1973).
4W.F. Brinkman, J. Serene, and P.W. Anderson, Phys. Rev. **A10**, 2386 (1974).
5D.N. Paulson and J.C. Wheatley, Phy. Rev. Lett. **41**, 561 (1978).
6N.D. Mermin and G. Stare, Report No. 2186, Cornell University, 1974.
7A.I. Sokolov and A.K. Tagantsev, Zh. Eksp. Teor. Fiz. **76**, 181 (1979) [Sov. Phys. JETP **49**, 92 (1979)].
8A.L. Korzhenevskiĭ and A.I. Skolov, Pis'ma Zh. Eksp. Teor. Fiz. **27**, 255 (1978) [JETP Lett. **27**, 239 (1978)].