Nonlinear waves in a medium with a semiconductor-metal phase transition

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The conditions under which a nonlinear temperature wave is formed in a medium with a semiconductor-metal phase transition are determined theoretically and a formula for their propagation rate is derived. An experiment, in which nonlinear waves in VO_2 films were observed, is described.

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The nonlinear waves in solids are a subject of constant interest. The discovery of the generation of uhf oscillations in semiconductors, pacticularly the Gunn effect, increased the interest in this field. The presence of a region with a negative differential conductivity (NDC) with a volt-ampere characteristic (VIC) is an important feature of the nonlinear effects. The states with NDC proved to be unstable[1] relative to the spatially inhomogeneous fluctuations. In semiconductors with an N-shaped characteristic such instability appears in the form of moving domains[2]; it was assumed[3] that for an S-shaped VIC the instability has the form of nonlinear temperature waves.

In this paper, we determine theoretically the conditions for formation of such waves in a medium with a semiconductor-metal phase transition (SMPT) and derive a formula for their propagation velocity. We also describe an experiment in which nonlinear waves were observed in VO_2 films.

A thin homogeneous film of a substance with a SMPT (see Fig. 1) with a width d, which is unlimited along the x axis, is located in a medium whose temperature T_A is held constant. A constant electric field E, which is directed perpendicularly to the x axis in the plane of the film, is produced in the film. The field produces a current, which flows in the film and heats it to a steady-state temperaure T. Let us assume that

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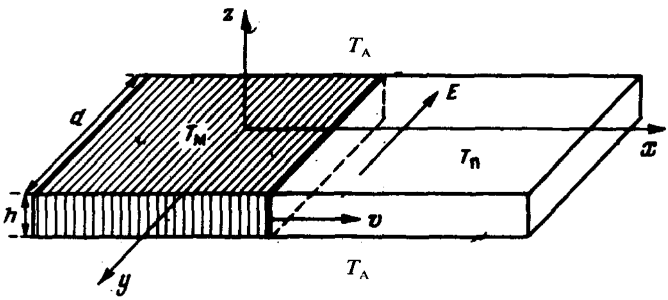

FIG. 1.

the temperature of the film is constant in its thickness and along the y axis. This corresponds to the thin-film approximation and to the condition that the heat transfer along the y axis can be neglected; the last condition is satisfied if $h/d \ll 1$, where h is the thickness of the film. All this indicates that the equation for the thermal balance in this model depends on one space variable and has a form

$$\rho C(T)\frac{\partial T}{\partial t} - \kappa\frac{\partial^2 T}{\partial x^2} = \sigma(T)E^2 - \frac{a}{h}(T - T_A). \tag{1}$$

Since the density ρ, the thermal conductivity κ, and the heat transfer coefficient α vary weakly as a result of the phase transition (PT), we ignore their temperature dependence. We examine the PT of the first order with the corresponding temperature dependence of the conductivity and of the specific heat:

$$\sigma(T) = \sigma_C + (\sigma_M - \sigma_C)\theta(T - T_0),$$

$$C(T) = C_1 + q_0\delta(T - T_0),$$

where σ_C is the conductivity of the semiconductor phase, σ_M is the conductivity of the metal phase, C_1 is the temperature-independent specific heat, q_0 is the heat of the phase transition, $\theta(T)$ is the theta function, $\delta(T)$ is the delta function, and T_0 is the temperature of the equilibrium SMPT.

Let us examine the homogeneous, steady-state solutions, then

$$\sigma(T)E^2 - \frac{a}{h}(T - T_A) = 0. \tag{2}$$

The roots of Eq. (2) determine the steady-state temperature of the film as a function of E and T_A. Figure 2 shows the dependence of the steady-state temperature T on the field E at a fixed ambient temperature. The dependence is S-shaped and for the fields $E_1 < E < E_2$ Eq. (2) has three steady-state solutions, two of which T_C and T_M are stable states of the system and T_0 is unstable.

Equations such as Eq. (1), whose right-hand side becomes zero at three points, were examined in Refs. 4 and 5. An asymptotic solution of Eq. (1), which describes the transition between the stable states, is a pair of diverging wave fronts. In the case of the PT the wave front describes the phase boundary. Let us find solutions in the form of

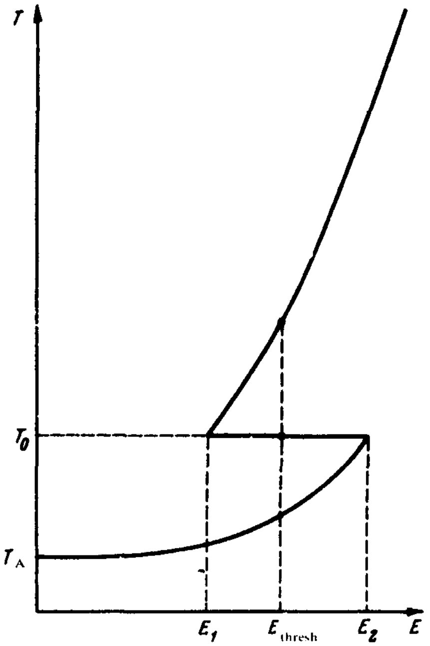

FIG. 2.

$T(x,t) = T(x - vt)$, where v is the velocity of the wave front. Going over to a new variable $f = x - vt$, we obtain

$$\kappa \frac{\partial^2 T}{\partial f^2} - v \rho C(T) \frac{\partial T}{\partial f} = \frac{a}{h} (T - T_A) - \sigma(T) E^2 . \tag{3}$$

The solution corresponding to the coexistence of two phases, i.e., $v = 0$, is possible only for a threshold field

$$E_{\text{thresh}} = \sqrt{\frac{2a}{h(\sigma_C + \sigma_M)} (T_0 - T_A)} \, . \tag{4}$$

For the case

$$\left| \frac{E - E_{\text{thresh}}}{E_{\text{thresh}}} \right| \ll 1$$

and $C_1(T_M - T_A) \ll q_0$, the expression for the velocity has the following form:

$$v = \frac{1}{\rho q_0} \sqrt{\frac{\kappa h^3}{a^3}} (T_0 - T_C^0)(\sigma_M^2 - \sigma_C^2) E_{\text{thresh}}^3 (E - E_{\text{thresh}}), \tag{5}$$

where T_A^0 is the steady-state temperature corresponding to the semiconductor phase at $E = E_{\text{thresh}}$.

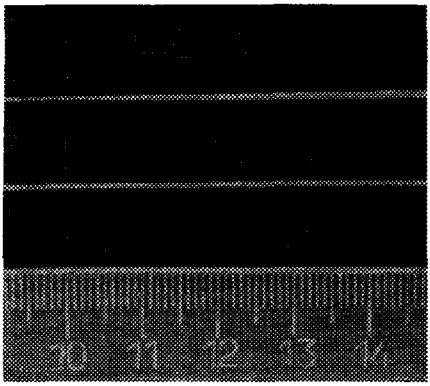

FIG. 3. The motion of a temperature wave in the VO_2 film. The dark area corresponds to the metallic phase. The photographs were taken at 2-sec intervals.

It can be seen in Eq. (5) that at $E > E_{\rm thresh}$ v is positive and corresponds to the propagation of the metallic phase and at $E < E_{\rm thresh}$ it changes its sign and corresponds to the propagation of the semiconductor phase. It can be seen, moreover, that the heat of the PT effectively reduces the velocity of the phase boundary. Note that the nonlinear wave exists only in the interval $E_1 < E < E_2$ (see Fig. 2). Experimentally we investigated VO_2 films on mica, in which a SMPT was observed at 68 °C with a conductivity jump of three orders of magnitude. The thickness of the films was 0.1–0.5 μm and the thickness of the substrate was 30–50 μm. Strips of aluminum electrodes spaced 0.5–1 cm apart were applied to the VO_2 film. A constant voltage from the B1–11 current supply was applied to the electrodes. The experiment was carried out under room conditions. The voltage of the sample, which was varied from 6 to 15 V, was chosen in such a way that the current passing through the semiconductor phase would not heat the sample to the phase-transition temperature. If the section of the film between the electrodes was externally heated to the phase-transition temperature so that the region of the metal phase would short circuit the electrodes, then, beginning with the voltages $E < E_{\rm thresh}$ the thermal wave front, which is the phase boundary, moved in the direction of the semiconductor phase with a constant velocity of several millimeters per second, which increased with increasing voltage. By varying the optical properties in the visible range, we were able to visually observe the motion of the temperature wave during the SMPT, (see Fig. 3).

In conclusion, we note that 1) the motion of the interphase boundary observed experimentally in the VO_2 film is in agreement with the discussed theory of motion of the nonlinear temperature wave; 2) the instability in the semiconductors with an S-shaped VIC shows that the homogeneous states are stable against small perturbations and that the spatially inhomogeneous perturbation has to be fairly strong for the nonlinear wave to occur; and 3) since the semiconductor-metal phase transition is accompanied by a variation of the optical properties, the nonlinear waves in the medium with the SMPT are of interest for the construction of space-time modulators of electromagnetic radiation.

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