Strong interaction of quarks as a mechanism of spontaneous symmetry breaking in the weak interaction

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A new mechanism of spontaneous symmetry breaking is proposed, which is connected with a negative contribution from the quark loops to the vacuum energy and with a strong interaction of quarks resulting in the reduction of this effect.

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Any gauge theory of weak interaction (for definiteness, we examine the standard $SU_2 \times U_1$ model with one doublet of the Higgs fields) must include spontaneous symmetry breaking due to the appearance of a non-vanishing average scalar field ϕ. In this paper, we describe a new mechanism of spontaneous symmetry breaking, which consists of the following components. The presence of fermions in the theory decreases the energy the vacuum with increasing scalar field [see Eq. (1) for the effective potential $V(\phi)$]. If we make the fermion-scalar field coupling constant sufficiently large compared to the gauge constant, then the theory will be unstable for large fields [curve III for $V(\phi)$ in Fig. 1]. For leptons this imposes a constraint on the corresponding Yukawa coupling constant, i.e., on the lepton mass. For quarks the critical factor is the strong interaction. At very high fields, in spite of the asymptotic freedom, the strong interaction of quarks with gluons becomes substantial and as result of which the fermion contribution to $V(\phi)$ dies out [see Eq. (4)]. The positive contribution to the potential from the interaction of the scalar field with the vector bosons causes the curve for $V(\phi)$ to rise. Thus, the gluons "dig a hole" whose depth and distance from the point $\phi = 0$ increase with decreasing color constant g_s. It is clear that for sufficiently small g_s the new minimum is the true equilibrium configuration of the system (if metastability is not allowed). It is also clear that the described effect can occur at both signs of the

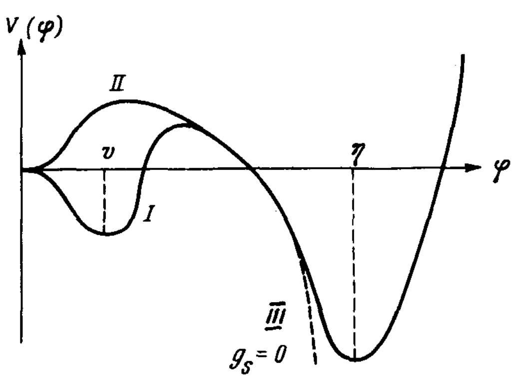

FIG. 1.

square of the original mass of the Higgs boson (curves I and II in Fig. 1). We show below that such a "gluon" mechanism of spontaneous symmetry breaking makes it possible under certain conditions to calculate the mass of the heaviest quark (it turns out to be ~60 GeV) and the mass of the Higgs boson (~7 GeV).

We write the expression for the effective potential $V(\phi)$ in a one-loop approximation, taking into account the loops of the W and Z bosons and of the heaviest fermion in the theory[1]

$$V(\phi) = \frac{1}{2} m^2\phi^2 + \frac{1}{4}\lambda\phi^4 + \frac{3}{2^9\pi^2}\left(g_W^4 - \frac{64}{3}h^4\right)\phi^4 \ln\frac{\phi}{\mu}. \qquad (1)$$

Here $g_W^4 = 2g^4 + (g^2 + g'^2)^2$, g and g' are the usual constants of the Weinberg-Salam theory, h is the Yukawa constant, and μ is an arbitrary normalization point whose overdeterminination changes the λ constant. We did not include in Eq. (1) the terms $\lambda^2\phi^4\ln\phi$, which is valid at $\lambda \ll g_W^2, h^2$. The most natural value of λ is $\lambda \sim g_W^4, h^4$, since λ varies by this order of magnitude as a result of variation of the normalization point. If the coefficient of the last term in Eq. (1) is negative, then we have a breakdown of $V(\phi)$ as $\phi \to \infty$. Thus, if the fermion under consideration is a lepton, then

$$h^4 < (3/64)\, g_W^4, \qquad M_l < \left[\frac{3}{4}(2M_W^4 + M_Z^4)\right]^{1/4}. \qquad (2)$$

At a Weinberg angle $\sin^2\theta_W = 0.23$, $M_l < 100$ GeV. For quarks we must include in Eq. (1) the interaction with gluons. Because of asymptotic freedom, we assume that $g_s^2 \ll 1$ but $g_s^2 \ln\phi/\mu \sim 1$, since for us the behavior of the potential is important at large fields. The quantity $V(\phi)$ can be determined either by directly analyzing the fermion loop in the external field ϕ by replacing the constnat h by the effective constant $\bar{h}(k)$ (the expression for $\bar{h}(k)$ is given in Ref. 2):

$$\bar{h}(k) = \frac{h}{\left[1 + b\frac{g_s^2}{8\pi^2}\ln\frac{k}{\mu}\right]^{4/b}}, \qquad b = 11 - \frac{2}{3}n_q, \qquad (3)$$

or by the method of renormalized group for the field ϕ.[1,3] In Eq. (3) n_q is the total number of quarks. As a result, we obtain the following expression for $V(\phi)$:

$$V(\phi) = \frac{1}{2}m^2\phi^2 + \frac{1}{4}\lambda\phi^4 + \frac{3}{2^9\pi^2}g_W^4\phi^4\ln\frac{\phi}{\mu} + 3h^4\phi^4\frac{1}{a g_s^2} \times\left[\left(1 + b\frac{g_s^2}{8\pi^2}\ln\frac{\phi}{\mu}\right)^{-a/b} - 1\right], \qquad a = 16 - b. \qquad (4)$$

As $g_s \to 0$, Eq. (4) becomes Eq. (1) with the factor 3 in front of h^4, which corresponds to the three colors of the quark. The equation for the minimum of $V(\phi)$ is

$$\frac{m^2}{\mu^2} e^{-2(x-1)\frac{8\pi^2}{g_s^2}\frac{1}{b}} + \lambda + \frac{3g_W^4}{16 b g_s^2}(x-1) + \frac{12h^4}{a g_s^2}\left(x^{-a/b} - 1\right) = 0, \tag{5}$$

$$x = 1 + b\frac{g_s^2}{8\pi^2}\ln\frac{\eta}{\mu}, \quad \eta = \langle \phi \rangle .$$

The first term in Eq. (5) can be dropped if there is a solution with $x > 1$ and m^2 is not too large (see below). We can drop λ in the remaining equation if $\lambda g_s^2 \ll g_W^4, h^4$. As already mentioned, we consider $\lambda \sim g_W^4, h^4$ a reasonable value when the condition $\lambda g_s^2 \ll g_W^4, h^4$ is satisfied. Therefore, we drop λ in Eq. (5). Thus, we can easily see that Eq. (5) has a solution when $x > 1$ if $h^4 > 1/64\, g_W^4$, i.e., when the inequality is opposite to that of Eq. (2) (taking into account the color factor). We assume that the original h constant, which is normalized for the field $\phi = \mu$, satisfies the required condition. The h constant, however, does not directly determine the mass of the quark Mq. The mass is determined by using the constant $h_q = \bar{h}(\eta)$, where $\bar{h}(\eta)$ is coupled to h by Eq. (3) in which k must be replaced by η. Equation (5), which is written in terms of h_q, is

$$\frac{\alpha(x-1)}{x(x^\alpha - 1)} = \frac{64 h_q^4}{g_W^4}, \quad \alpha = \frac{a}{b} = \frac{16}{b} - 1. \tag{6}$$

It is easy to see that at $x > 1 h_q^4 < 1/64 g_W^4$, i.e., h_q satisfies the opposite inequality compared to h. Hence, it is clear that the mass of the quark $M_q = h_q \eta$ is bounded from above analogously to Eq. (2):

$$M_q < \left[\frac{1}{4}\left(2M_W^4 + M_Z^4\right)\right]^{1/4} = 76 \text{ GeV} . \tag{7}$$

Equation (6), however, contains much more information, which makes it possible to calculate the mass of the heaviest quark if the value x is independently known. This value is determined by Eq. (5) in which $\eta = \angle(_F\sqrt{2})^{-1/2} = 259$ GeV and the point μ may be any suitable mass, for example, the mass of the c quark. [An ambiguity $\sim g_s^2$, however, exists due to the omission of λ in Eq. (5).] Thus, we have for the mass of the quark:

$$M_q = \left[\frac{1}{4}\left(2M_W^4 + M_Z^4\right)\right]^{1/4}\left[\frac{\alpha(x-1)}{x(x^\alpha - 1)}\right]^{1/4}, \qquad \alpha = \frac{15 + 2n_q}{33 - 2n_q},$$

$$x = 1 + b\frac{g_s^2(m_c)}{8\pi^2}\ln\left[\frac{1}{G_F m_c^2\sqrt{2}}\right], \qquad b = 11 - \frac{2}{3}n_q \cdot \tag{8}$$

Setting $(g_s^2/4\pi)\approx 0.2$, we find for $n_q = 6$, 8, and 10: $M_q = 61$, 59.5, and 58 GeV, respectively. As can be seen, M_q varies very weakly as a function of the number of quarks.

Equation (4) for the potential makes it possible to determine the mass of the Higgs boson $m_H^2 = (d^2V/d\phi^2)\phi_{\phi=\eta}$. A simple calculation gives

$$m_H^2 = \frac{3}{32\pi^2}[2g^2M_W^2 + (g^2 + g'^2)M_Z^2] - \frac{3}{\pi^2\sqrt{2}}G_F M_q^4 . \tag{9}$$

The first term coincides with the Coleman-Weinberg value[1] which they determined for $m = 0$. It follows from Eq. (9) and from the obtained values of M_q that for $n_q = 6$, 8, and 10: $m_H = 7.1$, 7.3, and 7.5 GeV, respectively. To achieve the described regime, we required that $h = \bar{h}(\mu) > (g_w/2\sqrt{2})^{1/2}$ without specifying μ. It seems that we can always achieve this if we choose a sufficiently small μ. It may turn out, however, that the first term in Eq. (5) cannot be neglected because μ^2 compared to m^2 is too small. The true condition of the "gluon mechanism" of symmetry breaking is that at $\mu = |m|$, $\bar{h}(m) > (g_W/2\sqrt{2})^{1/2}$. If this condition is not satisfied, then we have the usual situation in which the only minimum $V(\phi)$ is $v = \sqrt{-m^2/\lambda}$ (when $m^2 > 0$).

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