The $\pi\pi$-scattering phases from the analysis of the reaction $\pi^- p \to \pi^- \pi^+ n$ near the threshold

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The phases of the s_0 — and p_1 — wave $\pi\pi$ scattering near the threshold, which were obtained from an analysis of the reaction $\pi^- p \to \pi^- \pi^+ n$ in the energy range 200 MeV $< T_\pi <$ 260 MeV by using the Chu-Low method in the pseudoperipheral approximation, are given. The experimental data for the δ_0^0 phase confirm the theoretical prediction of the presence of the subthreshold zero in the s-wave $\pi\pi$ scattering amplitude.

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The main information on the phases of $\pi\pi$ scattering was obtained in recent years from the analysis of the reactions such as $\pi N \to \pi\pi N$ at incident pion momenta greater than several GeV/c.[1-4] The results of many studies are in reasonably good agreement in the region of the dipion mass 500 MeV $< m_{\pi\pi} <$ 1000 MeV. However, the amplitudes of the $\pi\pi$ interaction at low energies and small pathlength must be obtained by extrapolating the experimental data with the help of the effective-radius approximation from the region of $m_{\pi\pi}$ values relatively far from the threshold.

In this paper we analyzed 495 events of the reaction $\pi^- p \to \pi^- \pi^+ n$, which were obtained by the photographic emulsion method in the interval of kinetic energies of the incident pion 200 MeV $< T_\pi <$ 260 MeV.[5] This made it possible to determine the phases of $\pi^- \pi^+$ scattering in the range of the dipion masses from the threshold to $m_{\pi\pi}$ = 320 MeV.

To conduct the phase analysis, we used the technique of average spherical harmonics, which was described in Ref. 6. Since only the s and p waves are important in the radiation region, we can confine ourselves to the determination of two values $\sigma_{\pi\pi}$ and $\langle Y_2^0 \rangle$. The cross section of the $\pi\pi$ interaction was calculated in the pseudospherical approximation by extrapolating the function $F'(t) = F(t)/t$, which automatically satisfies the condition $F(0) = 0$. The use of the $F(t)$ function requires a quadratic extrapolation which produces unstable results in the case of insufficient statistics. By using the pseudospherical approximation we can reduce the power of the extrapolation polynomial[7] and confine ourselves to the linear extrapolation $F'(t) = A + Bt$, which gives stable results. The values $\langle Y_2^0 \rangle$ were also extrapolated to the pion pole according to the linear law. For the extrapolation we used the events with the square of the transferred momentum $t \geqslant 6\mu^2$. As follows from the calculations performed within the framework of the chiral theory,[8,9] the diagram of the one-pion exchange is one of the main diagrams for the $\pi N \to \pi\pi N$ reactions near the threshold.

To calculate the phases, we used the following reactions:

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$$\sigma_{\pi\pi} = \frac{4\pi}{q^2}\left\{\left(\frac{1}{3}\sin\delta_0^2 + \frac{2}{3}\sin\delta_0^0\right) + 3\sin^2\delta_1^1\right\},$$

$$\langle Y_2^0 \rangle = \frac{4\pi}{q^2\sigma_{\pi\pi}}\frac{3}{\sqrt{5\pi}}\sin^2\delta_1^1 . \qquad (1)$$

It can be seen from Eq. (1) that the δ_1^1 phase is determined directly and the δ_0^0 and δ_0^2 phases cannot be determined independently. Since we are interested primarily in the S_0 wave, we specified the values of the δ_0^2 phase in the calculation. To do this, we used the continuation of the threshold of the δ_0^2 phases which were determined in Ref. 10. Figure 1 shows the results for the $\sigma_{\pi\pi}$ cross section and for the δ_0^0 and δ_1^1 phases. A variation of the δ_0^2 phase in a wide range and also the use of other intervals of the subdivision according to $m_{\pi\pi}$ and t have a small effect on the results.

The investigated region of dipion masses has only the data obtained in the study of the K_{e4} decays. The points in Fig. 2 represent the difference in phases (δ_0^0 and δ_1^1) obtained in this paper and the crosses represent the difference in phases obtained in Ref. 11. As seen in Fig. 2, the results obtained from the different processes and by completely different methods are in good agreement.

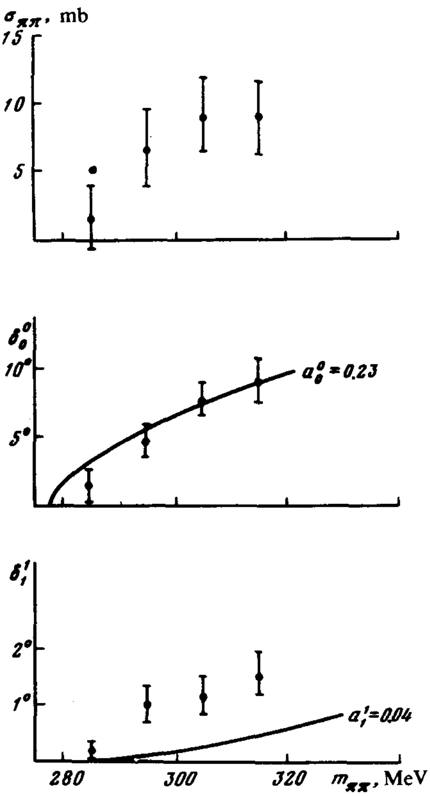

FIG. 1. The experimental values of $\sigma_{\pi\pi}$- of the δ_0^0 and δ_1^1 phases. The solid curves represent the calculation according to the theory with a broken chiral symmetry.

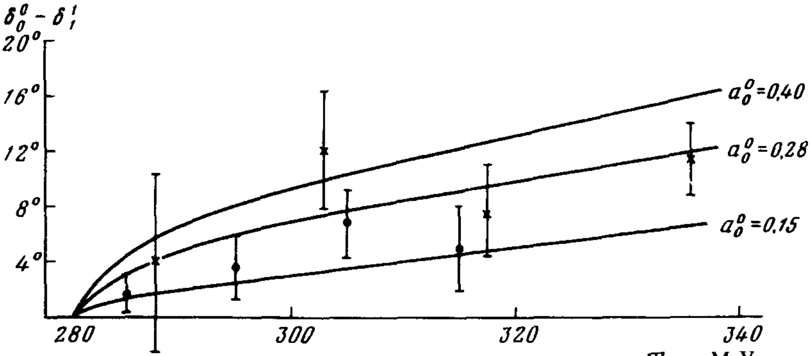

FIG. 2. The values of $(\delta_0^0 - \delta_1^1)$ obtained in this study (the points include errors) and in Ref. 11 (crosses). The solid lines represent the results of the calculations in Ref. 12.

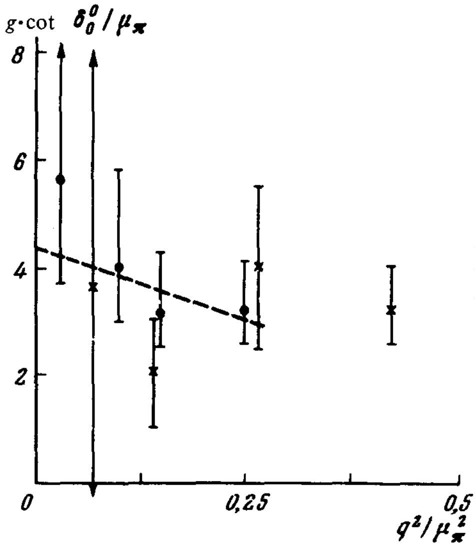

FIG. 3. The experimental values of q ctan δ_0^0 obtained in this study (●) and in Ref. 11 (✱). The broken line represents an approximation obtained by using the effective-radius formula.

To determine the scattering length a_0^0 according to the δ_0^0 phases, we used the effective-radius approximation which can be used near the threshold:

$$q\ \cot\ \delta_0^0 = \frac{1}{a_0^0} + \frac{1}{2} q^2 r_0^0 \,. \tag{2}$$

Here q is the momentum of the π meson in the center of mass system and a_0^0 and r_0^0 are the scattering length and the effective radius, respectively. The dashed line in Fig. 3 represents the experimental data approximated by Eq. (2). For comparison, we also give experimental points from Ref. 11.

We obtained the following parameters:

$$a_0^0 = 0.24 \pm 0.09 \ \mu^{-1},$$

$$r_0^0 = -9.6 \pm 19.1 \ \mu^{-1}.$$

The calculations of Basdevant *et al.*[12] for the different values of a_0^0 are in qualitative agreement with this result (the results of the calculations are represented by the solid lines in Fig. 2).

The value of a_0^0 is much smaller than $a_0^0 \sim 0.4 - 0.5 \ \mu^{-1}$, which was obtained by using the effective-radius formula in the extrapolation to the threshold of the δ_0^0 phases from the region $m_{\pi\pi} \geqslant 500$ MeV.[13-15] Garsevanishvili and Shirkov[20] and Franklin[21] showed that the effective-radius approximation in this region may give higher values of the scattering length a_0^0. We think that the result obtained by us is a direct experimental indication of the existence of a subthreshold zero in the S_0 amplitude of the $\pi\pi$ scattering.[16,17] An analogous conclusion was made in Ref. 18 on the basis of a combined analysis of the data from the K_{e4} decays[11] and the reaction $\pi^- p \to \pi^- \pi^+ n$ at $p_\pi = 17$ GeV/c.[2]

The data for the δ_0^0 phase obtained in this study are in good agreement with the predictions of the theory based on the broken chiral symmetry. The corresponding calculations[19] are represented by a solid line in Fig. 1.

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