Breakoff at constrictions in a current-carrying plasma pinch

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Analytic solutions are derived to describe the complete breakoff at long-wave constrictions in a plasma Z pinch with a current skin effect.

1. Book *et al.*[1] have numerically studied the nonlinear evolution of constrictions (the $m=0$ mode) in a pinch. This topic had been analyzed in the linear approximation by one of the present authors[2] in 1952. Although Book *et al.*[1] used an incompressible model, they were not able to study the complete breakoff at a constriction, since the situation $a_{\max}\to\infty$ was reached before the situation $a_{\min}\to 0$, and the numerical calculations could not be pursued. In the present letter we find an exact solution of the same equations as were used in Ref. 1, but our solution leads to the complete breakoff at the constriction. In our opinion, this breakoff is the most important phenomenon in a plasma pinch, and it must unavoidably be accompanied by a new, repeated breakdown of the current at the periphery of the constrictions, as has apparently been observed experimentally as a current filamentation.[3,4]

2. In contrast with Ref. 1, we use the model of a compressible gas for the pinch:

$$\frac{\partial\rho}{\partial t}+\operatorname{div}\rho\mathbf{v}=0,\qquad \frac{\partial\mathbf{v}}{\partial t}+(\mathbf{v}\nabla)\mathbf{v}=-\rho^{-1}\nabla p,\quad p\sim\rho^{\gamma} \tag{1}$$

with a pressure $p=B^2/8\pi=p_0(a_0/a)^2$ at the pinch boundary $r=a(z,t)$. In the long-wave approximation, we ignore changes in the functions p, ρ, and v_z over the plasma cross section, assuming $v_r=r(z,t)$. Under these conditions it is useful to introduce a dimensionless unknown function $A=(a/a_0)^m$, where $m=(\gamma-1)/\gamma$. System (1) along with the equation for the boundary then leads to the two equations

$$\frac{\partial}{\partial t}A^2+\frac{\partial}{\partial z}vA^2=0,\qquad \frac{\partial v}{\partial t}+v\frac{\partial v}{\partial z}=-c_0^2\frac{\partial}{\partial z}A^{-2}, \tag{2}$$

where $c_0^2=\gamma p_0/(\gamma-1)\rho_0=\gamma c_A^2/2(\gamma-1)$, and $c_A=B_0/\sqrt{4\pi\rho_0}$. Although system (2) describes the compressible model, it is precisely the same as that of the incompressible model of Ref. 1, except for the power-law dependence of A on the pinch radius a.

3. We have found that system (2) has a solution which is a particular solution but satisfactory in all regards and which can be found by a hodograph transformation. This solution can be written parametrically as (k and φ are parameters)

$$A=\Delta'^2/k,\ V=v/2c_0=(k'/\Delta')^2\sin\varphi\cos\varphi,\quad \pi c_0 t/\tilde\lambda=(E-K)/\Delta', \tag{3}$$
$$\pi z/\tilde\lambda=V\frac{\partial}{\partial\varphi}\Lambda_0-\Lambda_0,\quad \Lambda_0=KE(\varphi,k')+(E-K)F(\varphi,k').$$

Here $\tilde\lambda$ is the wavelength of the nucleating constriction, which we assume is given, as in Ref. 1; $K=K(k)$ and $E=E(k)$ are the complete elliptic integrals; $F(\varphi, k')$ and $E(\varphi, k')$ are the incomplete elliptic integrals, where $k'=\sqrt{1-k^2}$; and, finally, $\Delta'=\sqrt{1-k'^2\sin^2\varphi}$. In the limit $t\to-\infty$, solution (3) has the asymptotic form

$$A=1+\delta_t\cos Z,\quad V=-\delta_t\sin Z,\quad Z=2\pi z/\tilde{\lambda},\ \delta_t=8e^{T-2},\quad T=2\pi c_0 t/\tilde{\lambda},\tag{4}$$

which corresponds precisely to the linear approximation for a single harmonic with a given wavelength $\tilde{\lambda}$. Analyzing the nonlinear effects embodied in (3), we note that for the particular angle $\varphi=0$ we would have $A_{\max}=1/k$, while for $\varphi=\pi/2$ we would have $A_{\min}=k$, so that the time (T) dependence of $A_{\max}$ and $A_{\min}$ is determined by two different equations,

$$T=2[E(A_{\max}^{-1})-K(A_{\max}^{-1})],\quad T=2A_{\min}^{-1}[E(A_{\min})-K(A_{\min})],\tag{5}$$

from whose solution we find that just on the verge of the breakoff, in the limit $t\to-0$, we have $A_{\max}=\sqrt{-\pi/2T}$ approximately, while $A_{\min}=-2T/\pi$ ($T<0$). This result means that the maximum and minimum pinch radii evolve over time in accordance with

$$a_{\max}=a_0(-\tilde{\lambda}/4c_0t)^s,\quad a_{\min}=a_0(-4c_0t/\tilde{\lambda})^{2s},\quad s=\gamma/2(\gamma-1),\tag{6}$$

so that the situations $a_{\max}\to\infty$ and $a_{\min}\to 0$ arise simultaneously in the exact solution (3), in contrast with the numerical results of Book *et al.*[1]

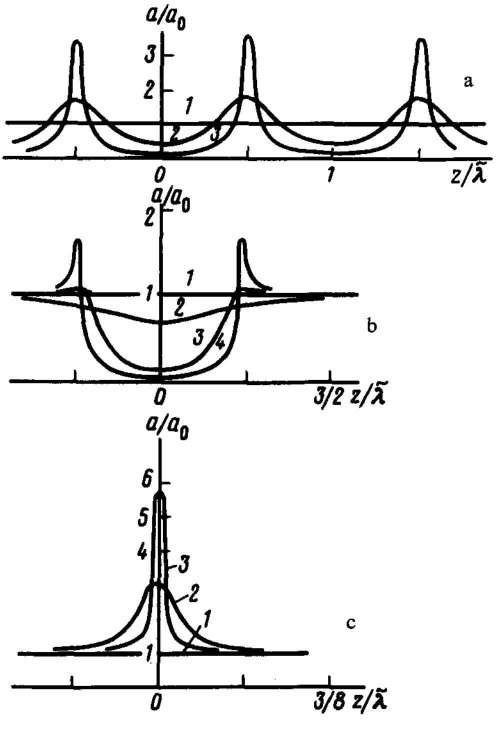

FIG. 1. a—Shape of the plasma boundary in the case of periodic constrictions; b—an isolated constriction; c—an isolated plasma ejection. Here $\tau=-\pi c_0 t/\tilde{\lambda}$. a—1–3: $\tau=\infty$, 0.314, 0.0628. b—1–4: $\tau=\infty$, 1.42, 0.142, 0.036. c—1–3: $\tau=\infty$, 0.142, 0.036.

4. Solution (3) corresponds to constrictions which are periodic along the pinch (Fig. 1a). An example of aperiodic but localized perturbations is described by another particular solution for which we have

$$-\pi z/\widetilde{\lambda} = 3\Lambda_0/2 + 3\pi/4 + \left[\left(\frac{1+k^2}{\Delta'^2} - 4 \right) H(k) - 3k'^2 K \right] \sin 2\varphi/4\Delta' ,$$

$$\pi c_0 t/\widetilde{\lambda} = H(k) \cos 2\varphi/2\Delta' , \tag{7}$$

$$H(k) = \frac{1+k^2}{k'^2} E - K,$$

while A and V are as given in (3). For the parameter values $\widetilde{\lambda} > 0$, $|\varphi| > \pi/4$ these relations determine an isolated constriction (Fig. 1b), while for the values $\widetilde{\lambda} < 0$, $|\varphi| < \pi/4$ they determine an isolated ejection of plasma (Fig. 1c).

The development of an isolated constriction, like that of periodic constrictions, culminates in complete breakoff. The time evolution of the maximum and minimum pinch radii is the same as in (6).

We evaluate the role of dispersion, determined by small terms $\sim (a_0/\widetilde{\lambda})^2$, in Ref. 5. Dispersion introduces the possibility that an isolated constriction will break up into seven smaller "subconstrictions," as is observed experimentally.[4]

5. Curiously, the possibility of a complete breakoff at constrictions has been ignored in the overwhelming majority of theoretical papers, and we believe that the approach is erroneous. It is the complete breakoff which must unavoidably cause, almost immediately (in a time on the order of a_0/c_A), an additional new current breakdown near the constriction through a new channel in the form of a hollow cylinder or discrete current filaments. This phenomenon is the "focus" which high-current pinches present us. After the repeated breakdown, most of the current comes to flow at the periphery, and the current in the residual central pinch decreases rapidly. When there is a cool, low-density plasma beside the central pinch, intense oscillations with frequencies on the order of the lower hybrid frequency may be excited near former constrictions, as we have shown elsewhere.[6–8]

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