Effect of rare collisions on plasma diffusion in stochastic magnetic fields

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We show that in a plasma with rare collisions electron diffusion across the magnetic field with perturbed magnetic surfaces is determined by a ratio of the mean free path and phase stochastization length in the magnetic field fluctuations.

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One of the possible causes of anomalously-high thermal conductivity of electrons in tokamaks is the presence of weak fluctuations of the radial component of the magnetic field.[1,2] The effect of these on the heat transfer process in a plasma were calculated in Ref. 3 in a collisionless case where the heat transfer is constrained by the destruction of magnetic surfaces due to fluctuations,[4] and in a hydrodynamic approximation.[5] However, the latter is no longer applicable to present-day tokamak plasma parameters and the conditions of a complete collisionless case have not been attained. In this connection we shall consider here the same problem in a weak collision approximation.

For the sake of simplicity we shall limit our considerations to a model of the magnetic field with plane magnetic surfaces and crossed lines of force in the presence of magnetic field fluctuations that are transverse with respect to the unperturbed magnetic surface

$$\mathbf{B} = B_z \mathbf{e}_z + B_y \mathbf{e}_y + \sum_{\mathbf{k}} B_{x\mathbf{k}}(x)\, e^{ik_y y + ik_z z}\mathbf{e}_x, \qquad B = |\mathbf{B}|. \tag{1}$$

To calculate the electron diffusion coefficient across unperturbed magnetic surfaces we shall use an equation for the electrons in the drift approximation which is averaged with respect to fast spatial variations of the magnetic field fluctuations:

$$\frac{\partial}{\partial t} \langle f_e(x, v_{\parallel}, t)\rangle = -\left\langle \sum_{\mathbf{k}} v_{\parallel} \frac{B^*_{x\mathbf{k}}}{B} \frac{\partial f_{e\mathbf{k}}}{\partial x} \right\rangle, \tag{2}$$

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where the brackets designate the aforementioned averaging, and $v_{\parallel}$ is the longitudinal particle velocity in the magnetic field. In contrast to the conventional quasi-linear equation, in our calculation of the fluctuating portion of the electron distribution function we take into consideration both the Coulomb collisions by way of a simplified collision integral in the BGK form,[6] and a nonlinear resonance broadening effect between the particles and magnetic fluctuations.[7] As a result of this, f_{ek} may be expressed as follows:

$$f_{e\mathbf{k}} = \left(-v_{\parallel}\frac{B_{x\mathbf{k}}}{B}\frac{\partial}{\partial x} + \nu_e \frac{n_{e\mathbf{k}}}{n_0} + \nu_{ee}\frac{m_e v_{\parallel}}{T_e} u_{\parallel\mathbf{k}}\right) < f_e > I_e(\omega, k_{\parallel}, v_{\parallel}), \tag{3}$$

where: $n_{e\mathbf{k}}$ and $u_{\parallel\mathbf{k}}$ are fluctuations of density and longitudinal hydrodynamic electron velocities, respectively, ν_{ee} and ν_{ei} are electron-electron and electron-ion collision frequencies, respectively, $\nu_e = \nu_{ee} + \nu_{ei}$,[8] and the value of I_e is determined by an integral over a perturbed particle trajectory with collisions taken into account (see Ref. 7).

$$I_e(\omega, k_{\parallel}, v_{\parallel}) = \int_{-\infty}^{0} d\tau \exp\left[\nu_e \tau + i k_{\parallel} v_{\parallel} \tau + \frac{1}{6} k_{\parallel}'^{2} D_{\perp} v_{\parallel}^2 \tau^3\right], \tag{4}$$

where

$$k_{\parallel}(x) = [k_y B_y + k_z B_z(x)]/B; \qquad k_{\parallel}' = \partial k_{\parallel}(x)/\partial x;$$

$D_{\perp}$ is electron diffusion coefficient whose dependence on the longitudinal velocity will be neglected since its inclusion in the final expression would only lead to appearance of a numerical coefficient of the order of unity. The averaged electron distribution function $\langle f_e \rangle$ may be considered locally Maxwellian assuming that a sufficiently large number of electron collisions occur during the diffusion. In this case Eq. (3) may be readily solved with respect to n_{ek} and $u_{\parallel\mathbf{k}}$ and the diffusion equation obtained upon integration of Eq. (2) with respect to velocities[1)]:

$$\frac{\partial}{\partial t} n_0(x,t) = \frac{\partial}{\partial x} v_{Te} \sum_{\mathbf{k}} \frac{|B_{x\mathbf{k}}|^2}{B^2 k_{\parallel}} \left\{\left[K_2 - \frac{i\nu_e}{k_{\parallel} v_{Te}} K_1^2 \Big/ \left(1 + \frac{i\nu_e}{k_{\parallel} v_{Te}} K_0\right)\right]^{-1} + \frac{2i\nu_{ee}}{k_{\parallel} v_{Te}}\right\}^{-1} \frac{\partial n_0(x,t)}{\partial x}, \tag{5}$$

where the functions $K_n = i k_{\parallel} v_{Te} \int_{-\infty}^{+\infty} (v_n/v_{Te})^n I_e(\omega, k_{\parallel}, v_{\parallel}) (< f_e/n_0) \, dv_{\parallel}$ in the linear case become conventional Crump functions Z_{II}, i.e.,

$$K_n \to Z_n(\zeta_e) = \pi^{-1/2} \int_{-\infty}^{+\infty} \frac{t^n \exp(-t^2)}{t - \zeta_e} dt, \tag{6}$$

where $\zeta_e = i\nu_e / k_{\|} v_{Te}$; $v_{Te} = (2T_e/m_e)^{1/2}$ is electron thermal velocity.

In view of the foregoing the integrals K_n may be calculated in the interval of interest to us $\zeta_e \gg 1$, $\nu_e^3 \gg k_{\|}'^2 D_\perp v_{Te}^2$. As a result of this we get the following expression for the diffusion coefficient:

$$D_\perp = D_{\|e} \sum_{\mathbf{k}} \frac{|B_{x\mathbf{k}}|^2}{B^2} \frac{\nu_{eff}}{k_{\|}^2 D_{\|e} + \nu_{eff}}, \tag{7}$$

where $D_{\|e} = v_{Te}^2/\nu_{ei}$ is the longitudinal electron diffusion, $\nu_{\text{eff}} = k_{\|}'^2 D_\perp v_{Te}^2 / \nu_e^2 \sqrt{\pi}$ is the effective frequency of collisions between electrons and magnetic field fluctuations. If we introduce of length of the electron phase in the fluctuations field $L_e \approx 1/\Delta k_{\|} = (D_{\|e}/\nu_{\text{eff}})^{1/2}$, Eq. (7) is reduced to the form obtained in Ref. 3

$$D_\perp \approx (D_{\|e}/L_e) \sum_{\mathbf{k}} \frac{|B_{x\mathbf{k}}|^2}{B^2} \pi\delta[k_{\|}(x)], \tag{8}$$

where, unlike in Ref. 3, the electron diffusion rate along the collapsed magnetic lines of force $\bar{v}_{\|} = D_{\|e}/L_e$ is determined not by the field length $L_0 \approx k_z^{-1}$ but by the particle phase length in the wave field. Solving Eq. (7) for $D_\perp$ we get

$$D_\perp \approx D_{\|e} b_0^4 \lambda_e^2 / L_x^2, \tag{9}$$

where $b_0^2 = \Sigma_{\mathbf{k}} |B_{x\mathbf{k}}|^2/B^2$, $\lambda_e = v_{Te}/\nu_e$ and $L_x \approx k_z/k_{\|}'$. When the inequality $\nu_{\text{eff}} < \nu_e$ is violated, this expression takes on the form obtained in Ref. 3 for the collisionless case, and when $\nu_{\text{eff}} < D/\delta^2$ (δ is the fluctuation correlation length along the x-axis), it becomes the same as the expression in Ref. 4. Thus, our "semi-collisionless" mode takes place in a sufficiently wide interval of the plasma parameters

$$L_0 (L_x / b_0 L_0)^{2/3} > \lambda_e > L_0 L_x / \delta. \tag{10}$$

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[1)]In fact one should speak of the electron thermal conductivity since the diffusion is always ambipolar due to occurrence of the electric field.

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