Theory of drift turbulence in a magnetic field with shear

A. S. Bakaĭ

Physicotechncal Institute, Ukrainian Academy of Sciences

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The interaction between the drift and convective modes leads to the formation of nonlinear wave packets–bound states of both types of waves, which are insensitive to the stabilizing effect of shear.

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In this paper we show that the well-known mechanism of stabilization of the drift-wave instabilities in the presence of shear is in effect when the intensity of the drift-convective turbulence reaches a critical value of W_c [see Eq. (10)]. The spectral density of the drift-convective turbulence at $W > W_c$ decreases according to the exponential law $n_k \sim \exp(-k^2/k^2)$ in the long-wave region $k\rho \ll 1$. Here we limit ourselves to examining the problem without regard for the toroidality. The equations of motion, which describe the interaction of the convective and drift modes in the magnetic field in the presence of shear, can be written in the following form[1,2]:

$$\frac{\partial n^d}{\partial t} - D_B \kappa_0 \nabla_\perp n^d - \rho^2 \frac{\partial}{\partial t}(\nabla_\perp n^d) - \frac{T_e}{m_i} \nabla_\parallel^2 \int^t n^d(t')\,dt' + \delta \hat{L} n^d = -\frac{D_B}{n_0}\left(\frac{\partial n^d}{\partial r}\nabla_\perp n^c - \nabla_\perp n^d \frac{\partial n^c}{\partial r}\right), \tag{1}$$

$$\left(\frac{\partial}{\partial t} - D_\perp \Delta_\perp\right)\Delta_\perp n^c = D_B \frac{T_e}{T_i n_0}\left(\nabla_\perp n^d \frac{\partial}{\partial r} - \frac{\partial n^d}{\partial r}\nabla_\perp\right)\Delta_\perp n^d, \tag{2}$$

where n^d and n^c are perturbations of the plasma density in the drift wave and in the convective mode, $D_B = cT_e/eB$, $\kappa_0 = n_0^{-1}\partial n_0/\partial r$, n_0 is the average plasma density, $T_{e,i}$ is the electron and ion temperature, m_i is the ion mass, c is the speed of light, $D_\perp$ is the transverse diffusion coefficient, and $\rho = c(T_e m_i)^{1/2}/eB$,

$$\nabla_\perp = (B_\phi/Br)\partial/\partial\theta - (B_\theta/BR)\partial/\partial\phi,$$

$$\nabla_\parallel = \frac{1}{qR}\left(\frac{\partial}{\partial\theta} + q\frac{\partial}{\partial\phi}\right), \qquad \Delta_\perp = \frac{1}{r}\frac{\partial}{\partial r} r \frac{\partial}{\partial r} + \nabla_\parallel$$

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B_ϕ and B_θ are the toroidal and poloidal field strengths, q is the safety factor for the stability, and R is the large radius of the torus.

The linear operator $\delta\widehat{L}$ in Eq. (1) takes into account the terms responsible for the instability of the drift waves, whose actual shape in this case is unimportant. We note that the short waves, for which $k\rho \leqslant 1$, as a rule, are the most unstable waves. As seen in Eq. (2), the build-up of short-wave drift oscillations leads to the excitation of long-wave convective modes, which in turn [see Eq. (1)] affect the stability of the drift waves. We seek a solution of Eqs. (1) and (2) in the form

$$n^d = e^{i(m\theta - n\phi)} \sum_{\nu=-\infty}^{\infty} a_\nu(r) e^{i\nu(l\theta - p\phi)} \tag{3}$$

$$n^c = b(r) e^{i(l\theta - p\phi)}, \tag{4}$$

where $l \ll m$, $p \ll n$, $m/n = q(r_{mn})$, $l/p = q(r_{lp})$, and r_{mn} and r_{lp} are the radii of the resonance surfaces in the neighborhood of which the oscillations are localized. For simplicity, we examine the case in which the convective mode has only one harmonic.

Substituting Eqs. (3) and (4) in Eq. (1), we obtain

$$\left\{\rho^2 \frac{\partial^2}{\partial x^2} + \left(\frac{c_s q'}{\omega q^2 R}\right)^2 [(m+\nu l)x - \nu l d]^2\right\} a_\nu$$

$$- \frac{D_B}{\omega n_0}\left[k_y\left(\frac{\partial b}{\partial x} a_{\nu-1} + \frac{\partial b^*}{\partial x} a_{\nu+1}\right) + \kappa_y\left(b\frac{\partial a_{\nu-1}}{\partial x} - b^*\frac{\partial a_{\nu+1}}{\partial x}\right)\right] \tag{5}$$

$$= \left[1 - \frac{D_B k_y \kappa_0}{\omega} + \rho^2 (k_y + \nu\kappa_y)^2 + \frac{\delta L(\omega, k)}{\omega}\right] a_\nu \equiv \Lambda a_\nu,$$

where $x = r - r_{mn}$, $d = r_{lp} - r_{mn}$, $k_y = m/r_{mn}$, $\kappa_y = l/r_{lp}$, and $\delta L(\omega, k)$ is the δL operator in the (ω, k) space.

We show that the convective waves weaken and at sufficiently large amplitudes (10) eliminate the stabilizing effect of shear on the drift waves. For this purpose, just like in Ref. 3, we substitute the differential equations in Eq. (5) for the difference equations and seek solutions in the form

$$a_\nu(x) = \exp[i\nu(\pi + \beta)]\, a(\xi),$$

where $\beta = \arg b$, $\xi = k_y x - \nu\kappa_y d$. As a result, we obtain the following equation for $a(\xi)$ with an accuracy to terms of the order of l/m:

$$\left(\mu \frac{\partial^2}{\partial \xi^2} + a^2 \xi^2\right) a(\xi) = \left(\Lambda + \frac{2 D_B k_y}{\omega n_0} \frac{\partial |b|}{\partial x}\right) a(\xi) \equiv \lambda a(\xi) \tag{6}$$

$$\mu = \rho^2 k_y^2 - \frac{D_B k_y \kappa_y^2 d}{\omega n_0}\left(2|b| - d\frac{\partial |b|}{\partial x}\right), \tag{7}$$

$$a = c_s r_{mn} q'/\omega q^2 R = c_s \theta / r_{mn}\omega .$$

(θ — shear).

We are interested in the eigenfunctions of Eq. (6), which vanish as $|\xi| \to \infty$. It is easy to show that the solution with the smallest eigenvalue of the modulus is the most unstable solution

$$\lambda = (-\mu a^2)^{1/2} . \quad (8)$$

This solution has the form

$$a(\xi) = A \exp(-a\xi^2/\sqrt{\mu}). \quad (9)$$

Since $|b|'d \sim |b|$, when $|b|$ is small, as seen in Eq. (7), $\mu > 0$ and the right-hand side of Eq. (8), which is purely imaginary determines the damping of the drift waves due to shear. The overwhelming influence of shear vanishes entirely at $\mu < 0$, i.e., at

$$|b| > b_c \approx \frac{\rho^2 k_y \omega}{D_B \kappa_y^2 d} n_0 \approx \frac{\rho^2 k_y^2 \kappa_0}{\kappa_y^2 d} n_0 , \quad (10)$$

when the right-hand side of Eq. (3) is real.

It can be shown that the obtained criterion is valid if we take into account the contribution of the electrons to the dispersion of the drift waves, which suppresses the drift instabilities.[4] As a result, the eigenfunctions (9) and the dispersion law (8) change slightly.

The drift waves in turn generate the convective modes, as can be seen in Eq. (2); this gives rise to nonlinear drift-convective oscillations which are insensitive to shear. The amplitude of the convective wave can be determined from Eq. (2):

$$|b| = \frac{2D_B k_y \kappa_y}{D_\perp (k_x^2 + \kappa_y^2)^2} \frac{T_e}{T_i n_0} \left| \sum_\nu [(k_y - \kappa_y) a^*_{\nu-1} a'_\nu + k_y (a^*_{\nu-1})' a_\nu] \right|$$

$$\kappa_x = b^{-1} \partial b / \partial x. \quad (11)$$

Substituting Eq. (9) in Eq. (11) we obtain

$$|b| = \frac{4\sqrt{\pi} D_B k_y^2 \kappa_y^2}{D_\perp (\kappa_x^2 + \kappa_y^2)^2} \frac{T_e}{T_i} \left| \frac{a^2}{\mu} \right|^{1/4} \frac{A^2}{n_0} . \quad (12)$$

In addition to the convective mode (l, p), the drift waves (3) give rise to all the harmonics $(\nu l, \nu p)$, whose amplitudes decrease exponentially with increasing ν. We should, therefore, expect that the spectrum of the drift-convective turbulence at small k $(k\rho \ll 1)$ has an exponential form: $n_k \sim \exp(-k^2/\bar{k}^2)$, $\bar{k}^2 = \sqrt{\mu}/ad^2$.

The obtained expression for the spectral density is in agreement with the exponential dependence obtained in Ref. 5 at a high level of turbulence. It is typical that in these experiments a broad k spectrum corresponds to each assigned value of ω, which indicates the presence of bound drift-convective waves.

The saturation amplitude of the drift turbulence can be easily determined from the equation for the energy balance:

$$\gamma_L = \gamma_s + D_\perp \kappa_y^2 (\kappa_x^2 + \kappa_y^2) | b |^2 / k_y^2 A^2,$$

$$\gamma_L = \mathrm{Im}(\delta L), \qquad \gamma_s = \mathrm{Im}[\omega (-\mu a^2)^{1/2}].$$

We note that the drift-convective turbulence leads to an anomalous transfer. The transfer coefficients can be estimated by using the available estimates of the turbulence level and the results of Refs. 6-8.

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[1]R.Z. Sagdeev, V.D. Shapiro, and V.I. Shevchenko, Fiz. Plazmy **4**, 551 (1978) [Sov. J. Plasma Phys. **4**, 306 (1978)].
[2]C.Z. Cheng and H. Okuda, Nucl. Fusion **18**, 587 (1978).
[3]J.B. Taylor, Plasma Phys. and Contr. Fusion Res. II. 323, LAEA, Vienna, 1977.
[4]L. Chen, P.N. Guzdar, R.B. White, P.K. Kaw, and C. Oberman, Phys. Rev. Lett. **41**, 649 (1978).
[5]A. Gondhalekar *et al.*, IAEA, Innsbruck, 1978, CN-37-C-4.
[6]A.A. Galeev, Problemy Teorii Plazmy (Theory of Plasma), Naukova dumka, 1973, Int. Symp. on Toroidal Plasma Conf., Garching, El-1, 1973.
[7]H. Furth and M.N. Rosenbluth, Plasma Phys. and Contr. Nucl. Fusion Res. **1**, 821, Vienna, 1969.
[8]A.S. Bakaĭ, IAEA, Innsbruck, 1978, CN-37-X-4-2.