Spin glasses with a semiconductor base

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An indirect exchange interaction between the localized spins in a semiconductor, which is produced by the conduction electrons, was calculated. It was found to be alternating and damping. The oscillation period and the damping constant are determined by the energy spectrum. The physical singularities are examined. The properties of spin glass with a quasi one-dimensional semiconductor base were studied. A comparison is made with the experiment.

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Abrikosov and Mukhin[1,2] examined spin glass with a short-range interaction, which can be produced by introducing into the metal in addition to magnetic impurities nonmagnetic impurities with a concentration that can satisfy the condition $n_m^{-1/3} \gg l$, where n_m is the concentration of the magnetic impurity and l and is the mean free path of the electron. It is rather difficult to realize such a situation (see Ref. 2). However, as we shall show, a very similar interaction between the localized spins occurs if they are located in a semiconductor rather than a metal.

We examine a model of a semiconductor with one maximum in the valence band at $\mathbf{p} = 0$ and several symmetric minima in the conduction band at $\mathbf{p} = \mathbf{K}_i$. The spectrum in the neighborhood of each extremum is assumed to be quadratic and isotropic with an appropriate effective mass. By measuring the energy from the ceiling of the valence band, we obtain the following Green's function:

$$G(\omega, \mathbf{p}) = \left[\omega + \frac{p^2}{2m_h} - i\delta\right]^{-1} + \sum_{\mathbf{K}_i} \left[\omega - \Delta - \frac{(\mathbf{p} - \mathbf{K}_i)^2}{2m_e} + i\delta\right]^{-1}, \tag{1}$$

where Δ is the energy gap.

If the interaction of the electrons with the localized spins is assumed to be in the form

$$\mathcal{H}_{ei} = -(J/n) \sum_i \psi^+(\mathbf{R}_i) \vec{\sigma} \mathbf{S}_i \psi(\mathbf{R}_i), \tag{2}$$

where $\mathbf{R}_i$ is the position of the localized spins $\mathbf{S}_i$, then we obtain an interaction between two impurity spins:

$$\mathcal{H}_{12} = 2(J/n)^2 \mathbf{S}_1 \mathbf{S}_2 \int \exp(i\mathbf{k}\mathbf{r})\, d^3k\, (2\pi)^3 [(-i) \int G(\omega, \mathbf{p}) G(\omega, \mathbf{p} - \mathbf{k}) \times d\omega\, d^3p/(2\pi)^4]. \tag{3}$$

We substitute in it $G(\omega, \mathbf{p})$ according to Eq. (1). In the integration over $\mathbf{p}$ we isolate only the special part (the other part is important only at atomic distances). If r is large,

we obtain

$$\mathcal{H}_{12} = -\frac{\pi^{-5/2}(am^*)^{3/2}}{[2(m_e+m_h)]^{1/2}}\left(\frac{J}{n}\right)^2 S_1 S_2 \frac{e^{-ar}}{r^{5/2}} \sum_i \cos \mathbf{K}_i \mathbf{r}, \tag{4}$$

where

$$a = \sqrt{2\Delta(m_e - m_h)}, \quad m^* = m_e m_h/(m_e + m_h). \tag{5}$$

At $\mathbf{K}_i \neq 0$ this interaction is similar to that of metallic spin glass with nonmagnetic defects. When $\mathbf{K}_i = 0$ (direct gap), we have a ferromagnetic interaction. A corresponding model was examined by Korenblit and Shender.[3]

The physical properties of glass were examined by Abrikosov and Mukhin,[1,2] who showed that at low temperatures the distanees r close to zero $\cos^2 p_0 r$ play the key role. The corresponding coefficient in the interaction (4) is $q = \Sigma_i \cos \mathbf{K}_i \mathbf{r}$, which depends on the band structure of the semiconductor. Let us define the density of the q values as follows:

$$\rho(q) = \lim_{V\to\infty} \frac{1}{V} \int d^3\mathbf{r}\, \delta\left[\sum_i \cos(\mathbf{K}_i \mathbf{r}) - q\right] \tag{6}$$

and determine it for two specific models, i.e., the Si and Ge type spectra. In the first case, we find

$$\rho(q) = \frac{1}{\pi} \int_0^\infty \cos qs\, J_0^3(\eta s)\, ds, \tag{7}$$

where $\eta = 1$ if the minima are located at the edges of the Brillouin zone and $\eta = 2$ if they are located at the common points of the fourfold axes. We can see that $\rho(0)$ is a finite value. This means that at low temperatures the temperature dependences of the specific heat and of the magnetic susceptibility are the same as those in the metallic glass with defects, which was examined in Refs. 1 and 2, i.e., $C \sim T$ and $\chi \sim \ln(T_0/T)$.

In the case of Ge type spectrum

$$\rho(q) = \frac{1}{\pi^3 \eta} \int_1^{4\eta/|q|} \frac{ds}{\sqrt{s^2-1}} K\left(\sqrt{1-\left(\frac{qs}{4\eta}\right)^2}\right), \tag{8}$$

where K is an elliptic integral and η has the same meaning as that in Eq. (7). At small q, $\rho(q)$ has the form

$$\rho(q << 1) \approx \frac{1}{2\pi^3 \eta}\left(\ln^2 \frac{32\eta}{q} - \frac{\pi^2}{4}\right). \tag{9}$$

This gives the following temperature dependences for the specific heat and magnetic susceptibility

$$C \sim T \ln^2(T_0/T), \qquad \chi \sim \ln^3(T_0/T), \tag{10}$$

where [we introduce the V_0 constant for $(\mathbf{S}_1\mathbf{S}_2)e^{-ar}r^{-5/2}\Sigma_i \cos \mathbf{K}_i r b(\psi)$]

$$T_0 = V_0 S n_m^{5/6} (4\pi n_m/a^3)^{2/3} \exp[-(4\pi n_m/a^3)^{-1/2}]. \tag{11}$$

These dependences differ noticeably from the preceding case. Above T_0 all the dependences are analogous to those in Refs. 1 and 2.

Abrikosov and Mukhin[1,2] assumed that the range of the forces l is much greater than the interatomic spacing; specifically, $l \gg (2p_0)^{-1}$. It can be shown that when $l \sim (2p_0)^{-1}$ or $\alpha \sim |\mathbf{K}_i|$ the theory remains valid in the range $T \ll T_0$.

All the discussed results are valid if the localized impurity spins can be located at any point in space. It is more natural to assume, however, that each impurity atom is located at a specific place in the unit cell of the crystal, although the cells themselves are arbitrary. If the oscillation period of the interaction forces is incommensurable with the lattice period, then therer is no difference. This holds for metallic glass in which the period is π/p_0 and for a semiconductor glass, if $\mathbf{K}_i$ is incommensurable with the reciprocal lattic periods. If, however, this is not so, then we have a special case—"a commensurable spin glass." A simple example is the models examined earlier, in which $\mathbf{K}_i$'s are situated at the edges of the Brillouin zone. If the magnetic impurities produce a substitutional alloy in a ZnS type lattice, then the model of the Si type spectrum with minima at the edges of the Brillouin zone has

$$\rho(q) = \frac{1}{4}\delta(q-3) + \frac{3}{4}\delta(q+1) \tag{12}$$

The model of the Ge type spectrum has four noninteracting spin subsystems in each of which

$$\rho(q) = \frac{1}{2}\delta(q-4) + \frac{1}{2}\delta(q+4). \tag{13}$$

All the temperature dependences in the commensurate spin glass retain their shape at $T \gg T_0$, but at $T < T_0$ there is a noticeable difference. Specifically, the magnetic susceptibility has a maximum in the neighborhood of T_0 (in addition to the maximum at Θ) and at lower temperatures it decreases with the temperature.

A spin glass with a quasi one-dimensional semiconductor base is an interesting model. The calculation, which is analogous to the three-dimensional case, gives the following formula for the interaction

$$\mathcal{H}_{12} = -\frac{4\eta}{\sqrt{2\pi}}\left(\frac{J}{n}\right)^2 \mathbf{S}_1\mathbf{S}_2 (m_e m_h)^{1/2} \cos Kz\, e^{-\alpha z}/(\alpha z)^{1/2} \tag{14}$$

where n is the number of atoms per centimeter of the chain, K is the location of the minimum in the conduction band, $\eta = 1$ at $K = \pi/a$, $\eta = 2$ in the other cases, and α is expressed by Eq. (5). The physical properties of such a material were examined in Ref. 4. In the one-dimensional model the commensurate case is identical to the noncommensurate.

In the case $K = 0$ we have a disordered magnetic substance with a ferromagnetic interaction. Such a material, which has not been examined before, has the following magnetic properties:

a) Magnetic susceptibility at $h \to 0$

$$\chi \approx \frac{2n_m \mu^2 S^2}{3T^{1+n_m/\alpha}} (V_0 n_m^{1/2})^{n_m/\alpha}, \qquad T \ll \Theta \sim V_0 n_m^{1/2} S^2 \tag{15}$$

$$\chi \approx \frac{n_m \mu^2}{3T}\left[S(S+1) + 2S^2 \frac{n_m}{a} \ln\left(\frac{V_0 n_m^{1/2}}{T}\right)\right], \qquad T \gg \Theta$$

b) Magnetization in the finite fields $1 \ll \mu hS/T$, $\lambda = (T/\Theta)^{n_m/a} \ll 1$

$$M \approx \frac{2}{3} \frac{n_m \mu^2 S^2 h}{T\lambda}\left(1 - \frac{4}{5}\gamma^2\right), \qquad \gamma = \frac{\mu hS}{T\lambda} \ll 1,$$

$$M \approx n_m \mu S (1 - 1/\gamma), \qquad \gamma \gg 1. \tag{16}$$

We measured experimentally the magnetic susceptibility of a semiconductor SrS with a small europium impurity. At an atomic concentration of less than 3%, Tolans observed the dependence $\chi \sim \ln(T_0/T)$ in the low-temperature region (Zh. Tolans, private communication). According to Eq. (11), T_0 depends on the concentration mainly according to the law where A varies slowly with n_m. The linear dependence $\ln T_0$ on $n_m^{-1/2}$ was confirmed experimentally. The value of α was estimated to be equal to $\sim 10^7$ cm^{-1}, which corresponds to Eq. (5). Unfortunately, a detailed comparison of the theory and experiment is difficult because the energy spectrum of SrS is not known in detail; moreover, in the experiments we used too large concentrations so that $4\pi n_m/\alpha^3$ is ~ 1 rather than $\ll 1$.

In conclusion, I would like to take this opportunity to thank Dr. Zh. Tolans for bringing my attention to this problem in the communication of his experimental results on $Eu_x Sr_{1-x} S$.

[1] A.A. Abrikosov and S.I. Mukhin, Pis'ma Zh. Eksp. Teor. Fiz. **27**, 477 (1978) [JETP Lett. **27**, 449 (1978)].
[2] A.A. Abrikosov and S.I. Mukhin, J. Low Temp. Phys. **33**, 207 (1978).
[3] I.Ya. Korenblit and E.F. Shender, Usp. Fiz. Nauk **126**, 233 (1978) [Sov. Phys. Uspekhi **21**, 832 (1978)].
[4] A.A. Abrikosov, J. Low Temp. Phys. (in press).