Localized nonlinear oscillations in ferromagnetic substances

B. A. Ivanov, A. M. Kosevich, and I. M. Babich
Physicotechnical Institute of Low Temperatures, Ukrainian Academy of Sciences

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An analytic solution, which takes into account the magnetic-dipole interaction, is obtained for a one-dimensional soliton of magnetization in a ferromagnetic. This solution, which corresponds to the periodic motion of magnetization in a soliton when its center of mass is stationary, describes in the limit the scattering of two domain walls.

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In recent years an interest has increased in the investigation of nonlinear localized magnetization waves in magnetic substances (magnetic solitons).[1] There is an exhaustive description of the dynamics of magnetic solitons in a one-dimensional model of a ferromagnet, which does not take into account the magnetic-dipole interaction based on the method of the inverse problem of the scattering theory.[2] It would be highly desirable, however, to take into account the magneto-dipole interaction and thereby to go outside the limits of such a model. This is particularly important because an omission of such an interaction greatly impoverishes the system, specifically by excluding the existence of moving domain walls.[3]

In this paper we obtain analytical solutions, which describe the magnetic solitons in a uniaxial ferromagnet taking into account the magnetodipole interaction. As far as we know, such as analysis was carried out earlier by numerical methods.[4]

Let us examine a plane magnetization wave that propagates perpendicularly to the "easy axis" (z axis). Using the usual condition for a ferromagnet $\mathbf{M}^2 = M_0^2$, we represent the magnetization vector $\mathbf{M}$ in the form

$$M_x = M_0 \sin\theta\cos\phi, \quad M_y = M_0 \sin\theta\sin\phi, \qquad M_z = M_0 \cos\theta. \tag{1}$$

If the direction of propagation of the wave is represented by the x axis, then the angle variables θ and ϕ are functions of x of the time t.

In dimensionless units (the unit length is equal to the thickness of the Bloch domain wall and the unit time is equal to the reciprocal frequency of the homogeneous ferromagnetic resonance in a system without a magneto-dipole interaction) the equations for θ and ϕ have the form:

$$\frac{\partial^2\theta}{\partial x^2} - \left[1 + \epsilon\cos^2\phi + \left(\frac{\partial\phi}{\partial x}\right)^2\right]\sin\theta\cos\theta + \frac{\partial\phi}{\partial t}\sin\theta = 0,$$

$$\frac{\partial}{\partial x}\left(\sin^2\theta\,\frac{\partial\phi}{\partial x}\right) + \epsilon\sin^2\theta\sin\phi\cos\phi - \frac{\partial\theta}{\partial t}\sin\theta = 0. \tag{2}$$

Equations (2) are the ordinary Landau-Lifshitz equations without dissipation, where ϵ is a parameter characterizing the ratio of the energy of the magnetodipole interaction to that of the anisotropy ($\epsilon > 0$).

Equations (2) have a solution that is localized in space and periodic in time:

$$\phi = \phi(t), \qquad \theta = \theta(x, t),$$

in which the $\theta(x, t)$ function vanishes at infinity ($x = \pm\infty$). This one-parametric solution has the form:

$$\tan^2\frac{\theta}{2} = \frac{1 - \Omega}{\Omega + \epsilon\cos^2\phi(t)}\,\frac{1}{\mathrm{ch}^2(x\sqrt{1 - \Omega})},$$

$$\tan\phi = \sqrt{(\Omega + \epsilon)/\Omega}\,\tan(\omega t + \gamma), \tag{3}$$

where the parameter Ω, which varies in the interval (0, 1), determines the frequency of the periodic motion $\omega = (\Omega(\Omega + \epsilon))^{1/2}$ and γ is an arbitrary initial phase. If we assume that $\epsilon = 0$, then $\phi = \omega t + \gamma$ and solution (3) goes over to solitons of nonuniform precession of the magnetization.[5]

In the limiting case $\Omega \to 1$ the frequency ω approaches the frequency threshold of the spin waves: $\omega \to (1 + \epsilon)^{1/2}$ and the amplitude of the soliton becomes vanishingly small in proportion to $(1 - \Omega)^{1/2}$.

More noteworthy is the limiting transition $\Omega \to 0$ ($\omega \to 0$). If the γ phase is arbitrary ($\gamma \neq \pi n$, n is an integer), then in the limit we obtain a solution which describes the stationary Bloch wall:

$$\cos\phi = 0, \qquad \tan\frac{\theta}{2} = e^{\pm x}. \tag{4}$$

If, however, $\gamma = \pi n$, then for finite times and $\Omega = 0$ we have

$$\tan\phi = \epsilon t, \quad \tan^2\frac{\theta}{2} = \frac{1 + \epsilon^2 t^2}{\epsilon}\,\frac{1}{\cosh^2 x}. \tag{5}$$

At $\Omega \ll \epsilon$ the plot of the function $\theta = \theta(x)$ periodically is shifted between the curve $\theta = \theta_{\min}(x)$ at $\omega t + \gamma = \pi n$ (curve I in Fig. 1) and the curve $\theta = \theta_{\max}(x)$ at $\omega t + \gamma = \pi n + \pi/2$ (curve II in Fig. 1). Curve I is determined by Eq. (3) in which $\phi = 0$ and $\Omega = 0$. The top of plot I corresponds to $\theta_{\min}(0) = 2\,\mathrm{arc}\tan(1/\sqrt{\epsilon})$. Curve 2, which differs greatly from curve 1, is described by the equation

$$\tan^2\frac{\theta}{2} = \frac{1}{\Omega\cosh^2 x},$$

from which it follows that in a wide range $\Delta x \sim |\ln\sqrt{\Omega}| \gg 1$ the $\theta_{\max}(x)$ function has a plateau $\theta \approx \pi$, which decreases sharply to zero in a narrow region ($\Delta x \sim 1$). Thus, the solution of Eq. (3) at $\Omega \ll \epsilon$ describes the oscillatory motion of two strongly interacting domain walls, which periodically move away from each other a distance of $\Delta x \sim |\ln\sqrt{\Omega}|$.

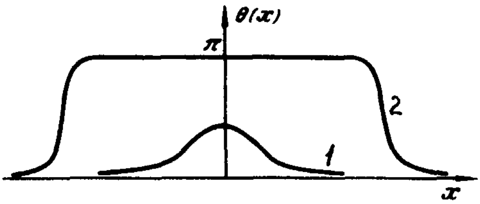

FIG. 1.

The limiting solution (5) is a special case of the aperiodic solutions for $-\epsilon<\Omega<0$:

$$\tan\phi=\sqrt{(\epsilon+\Omega)/|\Omega|}\tanh\left(t\sqrt{|\Omega|(\epsilon+\Omega)}\right),$$

$$\tan^2\frac{\theta}{2}=\frac{1-\Omega}{|\Omega|(\epsilon+\Omega)}\,\frac{|\Omega|+\epsilon\sinh^2\left(t\sqrt{|\Omega|(\epsilon+\Omega)}\right)}{\cosh^2(x\sqrt{1-\Omega})}\tag{6}$$

It is clear that as $t\to\pm\infty$ solution (6) describes two domain walls, which exactly correspond to the Walker solution and which converge (or diverge) with a constant velocity.[3] In fact, as can easily be seen, as $|\Omega|t\to\infty$ and $|x|\to\infty$ it follows from Eq. (6) that

$$\tan\frac{\theta}{2}=\frac{\sqrt{\epsilon}}{V}\exp\{-\sqrt{1-\Omega}(x-Vt)\},\qquad x>0,$$

$$\tan\frac{\theta}{2}=\frac{\sqrt{\epsilon}}{V}\exp\{+\sqrt{1-\Omega}(x+Vt)\},\qquad x<0,\tag{7}$$

where the velocity V (in dimensionless units) is

$$V=[|\Omega|(\epsilon+\Omega)/(1-\Omega)]^{1/2},\qquad -\epsilon<\Omega<0.$$

It is clear that Eq. (7) corresponds to two symmetrically diverging domain walls [$V=0$ corresponds to the special case (5)]. As $|\Omega|t\to-\infty$, we obtain two converging Bloch Walls. At the time $t=0$ the converging walls stop and then begin to diverge. At the moment of maximum convergence of the walls, we have

$$\tan^2\frac{\theta}{2}=\frac{1-\Omega}{\epsilon+\Omega}\,\frac{1}{\cosh^2(x\sqrt{1-\Omega})}.\tag{8}$$

It follows from Eq. (8) that at $\epsilon-|\Omega|\ll1$ the Bloch walls do not lose their individuality and the minimum distance between them is large $\Delta x\sim|\ln(\epsilon+\Omega)|$. If, however, $\epsilon-\Omega\gg1$, then the Bloch walls, in converging, become strongly deformed and lose their shape.

[1]A.M. Kosevich, B.A. Ivanov, and A.S. Kovalev, Sb. Nelineĭnye volny (Nonlinear Waves), Nauka, M., 1979, p. 45.
[2]M.L. Lakshmanan, Phys. Lett. **61A**, 53 (1977); L.A. Takhtajan, Phys. Lett. **64A**, 235 (1977); A.E. Borovik, Pis'ma Zh. Eksp. Teor. Fiz. **28**, 629 (1978) [JETP Lett. **28**, 581 (1978)].
[3]A. Hubert, Teoriya domennykh stenok v uporyadochennykh sredakh (Theory of Domain Walls in Ordered Media), Mir, M., 1977, p. 100.
[4]V.M. Eleonskiĭ, N.N. Kirova, and I.E. Kulagin, Zh. Eksp. Teor. Fiz. **74**, 1814 (1978) [Sov. Phys. JETP **47**, 946 (1978)].
[5]B.A. Ivanov and A.M. Kosevich, Zh. Eksp. Teor. Fiz. **72**, 2000 (1977) [Sov. Phys. JETP **45**, 1050 (1977)].